Massive black holes (MBHs) have been observed to reside in the centers of many (if not all) large and dwarf galaxies. Our own Milky Way is no different - it's MBH (called Sagittarius A*) has an estimated mass of approximately 4 million $M_{\odot}$ (where $M_{\odot}$ is the mass of the Sun). What's particularly interesting, there are many empirical relations which strongly suggest that MBHs are partly responsible for the processes governing their host galaxy evolution and vice versa. For instance, the amount of gas and stars available in the galactic reservoir may regulate the growth of the central MBH and this "feeding" rate in turn determines how much energy is released back (e.g. in the form of jets or winds, which can suppress star forming activity in the host galaxy).

Unfortunately, despite decades of studies, MBHs' birth and co-evolution with hosts are still two of the fundamental problems in present astrophysics. Especially, the discovery of luminous and extremely massive ($M > 10^8\ M_{\odot}$) quasars at the time when the Universe was less then billion years old revived discussions and research on how MBHs grow to these enormous sizes within such a short time. Addressing these problems is challenging due to the fact that the evolution process can start very early in the history of the Universe and our current observatories are unable to reach that far. What we can now do instead is to perform detailed simulations and by calibrating them to the available observational data derive constraints on the processes we cannot yet observe.

The foundation of most of such simulations is that galaxies and their central MBHs can successfully form binaries and merge in time shorter than the age of the Universe. An exciting consequence of such scenario is that a new messenger emerges - that is, we expect these systems to emit gravitational waves (GWs) and that we will be able to detect them. There are two major missions designed to search for such GWs. The first one, PTA (Pulsar Timing Array), is performing high precision millisecond pulsar observations in order to detect deviations induced by nanohertz GWs from the most massive ($M > 10^8\ M_{\odot}$) MBH binaries. The second mission, LISA (Laser Interferometer Space Antenna) is yet to start and designed to detect milihertz GWs from a complementary range of MBHs masses $M = 10^4 - 10^7\ M_{\odot}$.

In the consideration of all above, we have recently performed a pilot study using a novel, state-of-the-art model of galaxy evolution SHARK. It is an open-source, flexible framework which was successfully tested to match available observational data well and in addition to that, our preliminary results clearly show it can be a perfect tool to explore various scenarios related to MBH and galaxy co-evolution. We will therefore use SHARK in our project which we divided into three main objectives. First, we will develop a new model for MBH binary evolution as the default approach in SHARK doesn't yet account for processes taking place after the merger of galaxies. Then, we will run an extended set of simulations using various prescriptions of phenomena assumed to be directly linked to the MBH and host galaxy co-evolution. In the last part of the project we will compare our results with observations and provide predictions for future observatories. Particularly, we will focus on calculating the expected detection rates for LISA and estimating amplitude of the GW background generated by all MBHs in the frequency range targeted by PTA experiments. Given our new realistic population of MBH binaries we will also characterize their electromagnetic counterparts and compare them with observations of candidate systems where the existence of MBH binary is speculated but not yet confirmed. All of these results will let us impose new meaningful constrains on the processes involved in MBH and galaxy co-evolution and will pave the road to the future research on the nature and evolution of these exciting objects.