

## **Randomized numerical algorithms approximating solutions of ODEs**

Ordinary differential equations (ODEs) are the fundamental tool in mathematical modelling of many real-world phenomena. They play an important role in physics, chemistry, biology, engineering, finance and other sciences. Explicit solutions to the ODEs arising from practical applications are usually not known. Hence, advances in many scientific disciplines depend on the research in the field of numerical algorithms for differential equations. It is worth noting that randomized methods (which we deal with in this project) have, in comparison to deterministic methods, good numerical properties under very mild assumptions about the right-hand side function of the ODE, thus their applicability is broad.

Research in randomized numerical algorithms for ODEs is based on tools and results from several fields of mathematics, including real analysis, linear algebra, measure theory, probability theory, mathematical statistics and of course numerical analysis. Optimality of the algorithms is investigated in the context of Information-Based Complexity (IBC).

In this project we focus on the following problems: error analysis and optimality of the randomized algorithms under inexact information, stability of these algorithms and statistical view on randomized approximations to solutions of ODEs (e.g. confidence regions). These areas are currently intensively studied, however they are still not covered sufficiently in literature. We aim to solve the aforementioned problems for some classes of randomized algorithms.

We pay special attention to the optimal algorithms, i.e. such that their error (asymptotically) attains the minimal error (in certain sense) in the class of all algorithms satisfying some technical assumptions. Introducing informational noise in the model is important from the point of view of applications. For instance, inexact information framework can be used in mathematical description of lowering precision, which is an important topic in the context of efficient computations on graphics processors.

Stability is one of the fundamental properties investigated for deterministic numerical methods for ODEs. It tells whether the error of a given method, used to approximate the solution of an ODE with fixed step size, will be bounded in the long run. Surprisingly, for a long time this concept has not been adopted for randomized methods. In this project we use notions of probabilistic stability regions, which were introduced in the context of randomized algorithms for ODEs in our recent paper.

We also aim to construct bounds, between which the (unknown) exact solution of the ODE falls with pre-specified probability (so called confidence regions) and investigate properties of approximated values as point estimators of exact values. This important research area is still largely unexplored. We hope that our results will help to assess accuracy of the randomized algorithms, and facilitate effective simulation of trajectories of ODEs.

Impact of this project is expected to be two-fold. Firstly, theoretical findings will contribute to mathematical theory of numerical methods for ODEs and possibly they will set directions for further investigation. On the other hand, analyzed algorithms can be applied to problems of other sciences and to simulate real processes (e.g. models for spread of disease). There is a strong connection between theoretical and practical aspects since mathematically proven properties of numerical algorithms help to decide whether a numerical method is applicable or not for the given problem.