The main concern of the project is developing a fast algorithm that finds a winner in a given parity game.

What is a parity game? This is a game played on a directed graph by two players who are called Even and Odd. Every vertex of the graph belongs to one of the player, and is labeled by a positive integer called a priority. A play starts at a designated vertex and then the players move by following outgoing edges; it is always the owner of the vertex who moves by following an outgoing edge from the current vertex to the next one. The game is played forever, so at the end an infinite path is formed. In order to declare the winner we inspect the priorities of visited vertices. Namely, we check for the highest priority that occurs infinitely many times. Player Even wins if this priority is even, and otherwise player Odd wins.

Of course we cannot actually play parity games, since it requires an infinite amount of time. But we can try to "solve" a given game, that is, to find a player who is able to win—who has a winning strategy. We can also try to describe this winning strategy.

It turns out that parity games play a fundamental role in automata theory and logic, having applications to verification and synthesis. The algorithmic problem of finding the winner in parity games is polynomial-time equivalent to the emptiness problem for nondeterministic automata on infinite trees with parity acceptance conditions, and to the model-checking problem for modal mu-calculus. It also lies at the heart of algorithmic solutions to Church's synthesis problem. The impact of parity games reaches relatively far areas of computer science, like Markov decision processes and linear programming.

It is a long-standing open question whether parity games can be solved in polynomial time. For most algorithmic question we either have a fast (polynomial) algorithms, or we have a proof that no fast algorithm exists (NP-hardness). This is not true for parity games: it is still possible that a polynomial-time algorithm can be developed.

The goal of the project is to solve this problem: either show a polynomial time algorithm fining a winner in parity games, or to prove some hardness result matching the best known upper bound. While reaching this ultimate goal may be difficult, we hope for some partial results: improving the existing algorithms, as well as improving existing lower bounds. Moreover, beside of parity games, we want to investigate other games of similar nature: mean-payoff games, discounted-payoff games, and simple stochastic games; for these games we want to answer similar questions.

The project is directly motivated by a recent breakthrough algorithm solving parity games in quasipolynomial time (that is, in time $n^{\log n}$, which is not polynomial, but is quite close to polynomial), and by some results following it. Those recent results explained some aspects of parity games, but also introduced several new directions that one can try in order to fully understand the problem.

Research carried out in the project will be of a theoretical nature, it will mainly consist of inventing and proving theorems. In addition, we also want to write programs that implement some of developed algorithms.