Estimates of stochastic processes - probabilistic and geometric approach

Description for the general public

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Random variables are variables which values depend on outcomes of a random phenomenon. They are a basic object in the probability theory. A random variable can be the result of the coin toss or sum of the results in ten throws of a dice. A random variable can also be the price of an ounce of gold next Tuesday at 10. Often we are not interested in one particular random variable but in a set of them or more precisely - in the evolution of some random value in time. A much more interesting object than a random variable

$$X = \text{the price of an ounce of gold next Tuesday at 10},$$

is a stochastic process

$$X_t = \text{the price of an ounce of gold after time } t \text{ from now},$$

which gives us more information about the price of gold. A stochastic process is a collection of random variables $(X_t)_{t\in T}$ where T is the index set (often T is a subset of $[0,\infty)$ what can be interpreted as time). One of the most natural questions about the process X_t is: what is the biggest value of X_t? So: how expensive can gold be? We can write this as: how big can $\sup_t X_t$ be? Observe that $\sup_t X_t$ is a random variable. There is no universal criterion determining whether a random variable is "big" or "small". In this project we assume that $\sup_t X_t$ is big, if its expectation is big. It would be best if we calculated $\mathbb{E}\sup_t X_t$. Unfortunately this expression is often too complicated to do this explicitly. If so, we must estimate $\mathbb{E}\sup_t X_t$. This project aims at deriving certain "precise" (two-sided) bounds on $\mathbb{E}\sup_t X_t$ under certain assumption on the process $(X_t)_{t\in T}$. We will also try to obtain upper bounds which are easy to apply. To this end we plan to use Ramon van Handel's method of bounding stochastic processes. We will also use ideas developed by Michel Talagrand in his newest monograph.