Abstract for the general public

The diffusion equation describes processes of the spontaneous spreading out of particles, or the transport of energy (e.g., heat) they carry on, in some medium. Nowadays we know that it is a consequence of chaotic collisions of diffusing particles with themselves and with particles of the surrounding medium. In the school course of physics we learn about the classical example – it is the Brownian motion. The latter was discovered in the 20s of the XIXth century the Scottish botanist Robert Brown who, using an optical microscope, observed the motion of *Clarkia pulchella* pollen grains immersed in water. Brown noticed that these motions were irregular, jump-like, very fast, and seemed to appear without any influence of observable external force. Today we know that similar behavior can be seen when one looks at fat droplets embedded in milk and/or a pigment, e.g. ink, spreading in the solvent. Theoretical explanation of the Brownian motion was given independently by Albert Einstein (1905) and Polish physicist Marian Smoluchowski (1906). They both used, then just born, methods of statistical physics and pointed out that the random motion of pollen grains results from collisions between them and huge amount of the water particles which are much smaller than pollens, move very quickly and chaotically collide with the pollen grains leading however to some "net" effect observed as jumping motion. The Einstein and Smoluchowski theory became the first qualitative confirmation of the atomic hypothesis and explained basic properties characterizing a majority of diffusion phenomena. It provided the statistical physics based derivation of the basic equation describing the process – the so-called diffusion equation as well as justified the linear dependence between the mean square displacement of the diffusing particle from its initial position and the duration of the process. The standard diffusion equation works well but has two important disadvantages. The first has really the fundamental meaning – the propagation which it describes takes place with infinite velocity. This means that diffusing particles initially localized in a bounded region immediately spreads to the whole space – the probability to find a particle in an arbitrary distant point does not vanish. The second objection comes from the fact that in many complex system the relation between the mean square displacement and the time is not linear as it is rigidly required by the diffusion equation. In general it becomes, at least asymptotically for $t \to 0$ or $t \to \infty$, power-like. The above-mentioned stipulations force us to search for an alternative description of diffusion processes. Namely this aim constitutes the basic goal of my project which is oriented on introducing and analysis of the equation which governs the anomalous diffusion with finite propagation speed.

It is long-time known that the finite propagation velocity can be encoded adding the second time derivative to the evolution equation – the standard wave equation provides us with an example. Following this way in in the case of the diffusion equation we obtain the well-known telegrapher's equation involving both the first and the second order time derivatives, as well as an additional constant having dimension of time and related to the velocity through the diffusion coefficient The telegraphers equation has many practical applications - it describes the wave motion with damping, the signal loss in the transmission line, and so on. But the crucial for our purposes is that, being the hyperbolic equation, it automatically leads to the finite propagation speed. Thus the first of the above mentioned disadvantages disappears. But how to modify the second one, i.e. the linear dependence obeyed by the mean square displacement and time? It can be done by replacing the usual time derivatives by some expressions involving derivations and integrations – so-called integro-differential operators which physical sense is introducing the time non-locality, i.e., memory effects, to the equations governing the evolution. From the mathematical point of view such operators are called the (generalized) fractional derivatives. Connecting both approaches we arrive at the time non-local fractional equation which, if applied to the diffusion/heat transfer problems, is named the generalized Cattaneo equation. Physically meaningful solutions to this equation must satisfy a crucially important additional condition: to be acceptable as probability density distributions they must be normalizable and non-negative. Thus the specific research goal of the project reads: to find the conditions (suggested by phenomenological data, information on the medium structure and theoretical analysis of stochastic processes underlying the physical process) which, if required from the memory functions, guarantee that such solutions do exist.