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Project description

Many real life phenomena are controlled by rare events like natural disasters or crashes on the stock market. These unusual incidents are often juxtaposed with general trends of the typical events. This leads to an interplay between the rare and the typical events. The projects aims to develop new methods which would allow for an efficient analysis of such interplay. More precisely we will study probabilistic models which are constructed using components which exhibit two types of behaviour. The first one, which concerns typical components, is related to some kind of limiting behaviour of the whole model. The second one, which is exhibited by atypical components, leads to extremes among the components. If the model in question is sufficiently big the presence of the atypical components is unavoidable. In this case there is an interplay between contribution of typical and atypical components in some of the asymptotic aspects of the model. Usually one relies on the underlying limit theory if the typical components are dominant or the extreme value theory if the atypical components dominate. However there are no tools allowing to treat a scenario in which contribution coming from both types of components are comparable. Throughout this project we will develop some new methods suitable for analysis of this cases. To keep the approach illustrative, we will focus on specific models which exhibit a branching or bilinear structure.

The first type of models are tree indexed stochastic processes. More specifically we will study univariate and multivariate branching random walks and branching Lévy processes. We will investigate a system of evolving particles, where each particle follows a spatial motion. Additionally, at certain times, particles may split into a random number of new particles which then evolve independently form other particles present in the system. Such systems bear connections with Ising model on trees or Kolmogorov-Petrovsky-Piskunov-type partial differential equations. The connection to the latter one relies on the extremes of such systems. The underlying branching structure makes the number of particles present in the system to grow exponentially in time. This in turn, if the underlying spacial motion has a long range, makes the presence of atypical particles almost certain. One then sees an exponential number of typical particles and a constant order of atypical particles both of which contribute towards the asymptotic behaviour of the process. We aim to develop techniques which would lead to a description of frontiers of the system in the case when the contribution of typical and atypical particles are non-trivial. To the best of our knowledge these scenarios were not treated in general and there are no robust tools available in the literature.

The second type of models are bilinear functionals of long ranged multivariate stochastic processes. More precisely for two such processes, we will investigate ℓ_2-type scalar products and stochastic integrals. Functionals of this form appear most predominantly in financial mathematics. In this context stochastic integrals are used to model the evolution of prices of given assets. Despite its bilinear nature the only tools available in the literature allow to treat the case when one process dominates the other, making the functional and its asymptotic linear in practice. We aim to develop a theory for bilinear functionals which would to allow to experience truly bilinear functionals, i.e. those with a comparable contribution coming from both processes. This structure makes the asymptotic of the functional in question bilinear as well.