The main problems studied in the present project concern properties of Calabi-Yau manifolds belonging to the most studied objects in algebraic geometry, a branch of mathematics connecting methods and objects of algebra and geometry.

Interest in Calabi-Yau manifolds is threefolds is inspired by applications in theoretical physics. The superstring theory predicts that the universe is not described by the four variables of the usual Minkowski time-space, but rather with ten parameters. Additional six parameters (or three complex parameters) describe a Calabi-Yau threfold, i.e. complex manifold of dimension three satisfying certain conditions that can be expressed in terms of differential geometry (the holonomy group is contained in the group $SU(3)$) or algebraic geometry (trivial canonical bundle and vanishing the first Betti number).

Calabi-Yau manifolds of dimension greater than 3 are significantly less studied, the only classes of examples available in an arbitrary dimension are constructed as resolution of singularities of a quotient of a product of varieties of lower dimension by an action of a finite group and a double covering of the projective space $\mathbb{P}^n$ branched along an arrangement of $2n + 2$ hyperplanes. Calabi-Yau manifolds of the double cover type are little studied, we do not know an exact classification of admissible types of singularities of arrangements of hyperplanes, there are no formulas for the Hodge numbers, which are the most important invariants of Calabi-Yau manifolds (significantly finer than the Betti numbers).

Similarly as in the case of elliptic curves, which are Calabi-yau manifolds of dimension one, in the case of an arbitrary dimension n we can define period integrals as integrals $\int_\gamma \omega$ of the canonical form ω over n-cycles $\gamma \in H_n(X, \mathbb{Z})$. Although the period integrals are analytical objects, they are closely related to arithmetic properties of Calabi-Yau manifolds expressed by the modularity theorem theorem, the higher-dimensional counterpart of the Taniyama-Shimura-Weil Conjecture. The modularity theorem is proved only in the special case of of Calabi-Yau manifolds with $b_n = 2$ defined over the field of rational numbers.

The first examples of Calabi-Yau threefolds with a small absolute values of the Euler characteristic (predicted in the string theory) were constructed as small resolutions of threedimensional hypersurfaces of degree 5 with ordinary double points (the so-called nodal quintics). The classification question of nodal quintics still remains completely open. A surprising feature of threedimensional nodal quintics is that their invariants depend not only on the number but also on the position of nodes expressed by the defect. The construction of a small resolution is of analytical nature, deciding whether a nodal three-dimensional variety admits an algebraic (projective) small resolution is usually very difficult and requires determination of smooth surfaces generating the Picard group of the considered variety.

A spectacular achievement of algebraic geometry in 20th century is the discovery of a connection between arithmetic properties of algebraic varieties and analytic properties of certain differential equations. A one parameter family of Calabi-Yau manifolds Y_t of dimension n has an associated differential equation called the Picard-Fuchs equation, annihilating a period integral $t \mapsto \int_{\gamma_t} \omega_t$, where $\gamma_t \in H_n(T_t, \mathbb{Z})$. In the case of a family of threedimensional manifolds, this is an ordinary linear differential equation of order 4, i.e. an equation of the form

$$P_4(z)f^{(4)}(z) + P_3(z)f^{(3)}(z) + \cdots + P_1(z)f'(z) + P_0(z)f(z) = 0.$$

In this project we propose to study the following problems

- find formulas for Hodge numbers of Calabi-Yau manifolds of double octic type, study natural elliptic fibrations on Calabi-Yau manifolds of double cover or generalized Kummer type, generalize the construction of fiber products of rational, jacobian elliptic surfaces to the case of a fiber product of elliptic fibrations over a higher dimensional variety or higher dimensional Calabi-Yau manifolds over a projective line $\mathbb{P}^1$,

- study relations between period integrals of singular elements of one parameter families o Calabi-Yau manifolds with special values of modular forms, construction of non-rigid modular Calabi-Yau threefolds, identification of entries of monodromy matrices of Picard-Fuchs operators, application of analytic methods from the differential equations theory to study modularity of Calabi-Yau manifolds with $b_n(X) > 2$,

- application of combinatorial methods for arrangements of planes in $\mathbb{P}^4$ to classification of nodal quintics and to construction of universal deformation, determination of the Hodge numbers of nodal degenerations in certain classes of Calabi-Yau manifolds.