

Random processes in random environment

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Random discrete structures have gained in importance in recent years. This is an area of probability, full of interesting, often surprising, results, which connects mathematics with theoretical computer science and statistical physics. In the framework of the project we intend to consider fundamental random objects such as random walks, branching structures, urns models and others. All of them have been intensively studied for decades, however we will observe them from a new point of view, which is still not well understood and study their behavior in some additional random environment. For many practical reasons one should consider models where the medium in which the process lives is irregular and reflects random factors like defects, fluctuations.

A simple random walk on the set of integers can be described as a random motion of a particle, initially placed at zero, which always moves one step left or right with probability $1/2$. Thanks to a natural definition and a relatively simple structure, this process for a long time has been used to model numerous random phenomena. Its behavior is quite well understood because the random walk can be represented as a sum of independent and identically distributed random variables. However the homogeneity of the environment where the system evolves may be unrealistic. In many practical cases some kind of obstacles can appear like impurities, fluctuations, etc. It is natural to model such irregularities as a random environment, which affects behavior of the random walk.

Branching processes model biological reproduction processes, but thanks to their tree structure found applications in various fields, including e.g. computer science and physics. The branching process usually starts with one individual, who produces offspring according to some random rule. The collection of offspring constitutes the first generation. Each of these individuals has children with the same progeny distribution, independent of the others and of the parent. This process continues forever or until the population dies out. In many cases it may happen that its evolution conditions change randomly over time, therefore one considers branching process with random offspring distribution.

Broadly speaking the main research objective of the project is the analysis of properties of fundamental random objects in random medium, emphasising the role of the additional randomness. On the one hand we want to deeply understand how random environment influences some of the properties of the random process. Although this problem has been already studied in the literature and it is known that adding randomness to the environment leads to new, often unexpected phenomena, numerous significant questions are still open. Above all else we would like to understand to what extent the impact of the random environment is essential. By perpetuating the medium we want to investigate fundamental properties of the process and to localize the moment, when the randomness of the process itself, not the environment, begin to play a dominating role. We hope this will lead to better understanding of random processes in random environment.