Controllability of systems of PDEs

(abstract for the general public)

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Partial differential equations (PDEs) are widely used as models of many physical phenomenons. They are classified into several types, two of the most important being "parabolic" and "hyperbolic". Both types of equations evolve in time and space. For instance, modeling how a quantity such as heat diffuses through a given region leads to a parabolic PDE, the heat equation, whereas describing the propagation of water in open channels gives a hyperbolic system of PDEs, the celebrated Saint-Venant equations.

A question of great interest for both applications and theory is to know whether a PDE is "controllable". For instance for the Saint-Venant equations, this amounts to answer the following question: is it possible to regulate the water flow in some open channel by means of some devices like spillways in order to reach, after some time, a desired flow rate determined beforehand ? The generic name for such devices is "control" since their goal is to control certain quantities of the equations.

The objective of this research project is to investigate what happens for the controllability of **systems** of PDEs, that is when we consider not anymore one single PDE but many of them that are interacting together.

Mathematical tools currently used to investigate the controllability of a given PDE do not always work for systems or only yield partial results. In this research project we will try to shed light on what happens for systems by developing techniques to address this issue. We will first investigate the controllability of two toy models of parabolic and hyperbolic systems and tackle specific problems such as for instance: what is the best time in which the controllability is feasible ? or what is the minimum number of controls needed ? Answers to such questions are essential for numerous applications. Depending on our findings we will then aim for generalizations.

We believe that our investigation will contribute to better understanding of the controllability properties of systems of PDEs.