Fatou theorems for harmonic functions and mappings and their generalizations
(popular science abstract)

Harmonic functions belong to the area of PDEs and the related Laplace equation is one of the fundamental equations in mathematics, a base for various other types of elliptic and parabolic PDEs. Mathematical models relying on harmonic functions appear, for instance, in heat flow theory (temperature distribution model), fractal theory, astrophysics, description of the glaciers dynamics, as well as in quantum mechanics.

A notion of harmonic function has been considered in various contexts and spaces. One of the main objectives of our project will be the settings of manifolds and the Heisenberg-type groups. Roughly speaking, the manifold theory investigates the geometry of objects with curvature, for example, surfaces, spheres, tori. Moreover, the studies of curvature of the space are one of the cornerstones of the General Relativity Theory and modern investigations of the universe and its models. As for the Heisenberg groups, they serve as an important class of noncommutative groups based on the Euclidean spaces. The structure of such groups leads to interesting notions of distance and measure and one of their intriguing applications is a model of the visual cortex.

Main goal of the project is to study the boundary behaviour of harmonic functions. We will pursue answers to the following questions: how big can be the oscillations of a harmonic function at points in the vicinity of the boundary? What can be said about the geometry of the boundary basing on the oscillations, also do they characterize some types of domains? These questions will be studied in domains on manifolds and in the Heisenberg-type groups. Similar questions will also be considered for p-harmonic functions, an important generalization of the harmonic functions.

Another direction of generalizing harmonic functions are vector-valued functions. Such a generalization leads to the notion of harmonic and p-harmonic mappings which are related to the calculus of variations and modelling of the solid deformations in space. It turns out that the problem of finding a deformation with least energy often leads precisely to harmonic mappings or their appropriate generalizations. We will be investigating the boundary behaviour of such mappings. In particular, our goal is to understand the structure of the boundary points, where mappings posses limits in the appropriate sense. Furthermore, for such a set of points, we will study relations between its geometry and the geometry of the underlying domain.

The results of the project will deepen understanding of the boundary behaviour of harmonic functions and their generalizations, especially for domains in curved spaces and on certain types of noncommutative groups. Our investigations will require developing the appropriate techniques in harmonic analysis and the growth estimates near the boundary of domains. Moreover, our studies will deepen understanding of the mapping theory.