SOLUTIONS OF THE QUANTUM YANG–BAXTER EQUATION AND ASSOCIATED ALGEBRAIC STRUCTURES: (SEMI)GROUPS, ASSOCIATIVE ALGEBRAS AND SKEW BRACES

Popular science summary

The project is concerned with algebraic methods and structures created in the study of the quantum Yang–Baxter equation. This equation, nowadays one of the most important equations in mathematical physics, has laid foundations for a rapid development of a number of areas of mathematics, including theory of quantum groups and certain aspects of the theory of Hopf algebras. The quest for solutions of this equation, as well as its several applications, has been accomplished to a large extent via developing algebraic tools, based on theory of groups and semigroups, theory of associative algebras and other combinatorial and algebraic structures, including the so-called braces and their generalization. The fundamental open problem motivating the project was posed in 1992 by Drinfeld and it concerns the description and classification of all the so-called set-theoretic solutions of the quantum Yang–Baxter equation. The proposed research methods are based on groups, semigroups, associative algebras (including algebras related to the braid relation, that has already played an eminent role in several areas of mathematics) as well as on braces and their generalizations (especially the so-called skew braces).

It is known that the so-called structure groups, monoids and algebras of solutions of the Yang–Baxter equation encode a lot of information about the solutions themselves. On the other hand, skew braces (a skew brace is a set equipped with two group structures that are linked by a substitute of the distributivity law) lead to a very broad class of solutions of the Yang–Baxter equation. Therefore, the fundamental problem in this area is to investigate structural properties and representations of such objects (that is groups, monoids and algebras associated with solutions of the Yang–Baxter equation and skew braces), and then to understand the intimate interplay between algebraic properties of such objects and combinatorics of solutions of the Yang–Baxter equation. Moreover, since skew braces are generalizations of both groups and nilpotent rings, it is possible that the research techniques developed within the project will shed a completely new light on certain old open problems in group and ring theory (e.g., Kaplansky's conjecture on units in group rings or Köthe's conjecture concerning nil-ideals in rings).

The motivation and the starting point for this project come from recent results of many authors, that are on the crossroads of the theory of groups, theory of semigroups, structural and combinatorial ring theory, representation theory, theory of quantum groups and Hopf algebras, knot theory and some aspects of mathematical physics. The fact that this is an area of current interest of a wide group of researchers is, in particular, evidenced by a number of publications in leading mathematical journals, as well as more then 180 preprints related to the Yang–Baxter equation posted in section "mathematics" on arXiv during the last three years.