Quantum groups, graphs and symmetries via representation theory

Adam Skalski and Moritz Weber

In the proposed research project, we intend to investigate the phenomenon of quantum symmetry from various angles: representation theory, combinatorics, operator algebras and quantum information theory. We will push forward our detailed understanding of quantum groups and their intertwiner spaces; we will provide new examples of such objects on the one hand and new quantum channels for quantum information theory on the other; and we will study the very recent new concept of quantum graphs.

Let us briefly sketch the background of our proposal. Symmetry is ubiquitous in mathematics and science. Mathematically, this is treated by investigating actions of groups. However, in modern branches of mathematics and physics, we need a more general concept of symmetry, namely quantum symmetry. On the mathematical side, we shall now employ quantum groups as the correct formalization, as introduced in the 1980s; we will focus on the analytic definition given by the famous Polish mathematician Stanisław Lech Woronowicz.

One of the main tools for studying quantum groups is their representation theory, or slightly more technical: their intertwiner spaces. The knowledge here is already quite advanced, and yet at the same time finer details remain known only in very special cases. In our first strand of research, we plan to close this gap for the large class of so called "easy" quantum groups or partition quantum groups. They have been defined about a decade ago admitting a quite combinatorial and diagramatic treatment.

In our second strand, we will study a generalization of graphs. Graphs, in a mathematical sense, are used at many instances in science. Recently, a generalization to quantum graphs has been developed in several steps. These objects are far from being understood and we will study their quantum symmetries, their interplay with operator algebras and we will associate new objects to them.

Finally, our third strand deals with quantum channels. These are the means to transport information in quantum information theory. Building on recent constructions, we will develop and study new quantum channels arising from partition quantum groups, we will analyse their entropy production and we will work towards a conceptual understanding of their interplay with quantum groups and tensor categories.

The research will be undertaken by a Polish team at Warsaw and a German team at Saarbrücken. There has been a fruitful exchange between these two nodes in the past and it is now time to work together in a more systematic way. Complementing our expertise, we will jointly work on all of the above problems. Besides, this will strengthen the ties between our institutions and build the ground for further future collaborations.

Specialists working on quantum groups, operator algebras and quantum information theory will profit from our interdiciplinary approach. Eventually, this will expand our general knowledge on quantum phenomena from a mathematical perspective.