# Generalizations and applications of Białynicki-Birula decomposition

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A wonderful fact is that the laws of physics tend to be independent of the coordinates chosen: experiments repeated today yield the same results as yesterday ones, laws of classical mechanics do not depend on coordinate systems, Maxwell's equations are invariant under Lorentz transform (which was of importance in kick-starting the special relativity). These phenomena are by no means obvious: classical theoretical frameworks allow for very coordinate-dependent models.

The same phenomenon holds in mathematics and in particular in geometry. Here we view our system as a shape or variety and the invariant changes of coordinates form a set of transformations on the variety called the *group action*. Again, usually the "general" varieties permitted by the theory tend to have little to no group action while the varieties "arising frequently" tend to have large group actions.

Having a variety with a group action, we can identify the point with its image under any transformation, for example if we act on $\mathbb{R}$ by translation by 1, we identify 0 with 1 with 2 etc, so we get the circle $\mathbb{R}/\mathbb{Z} = S^1$. This identification frequently makes the variety easier to understand, for example it can reduce its dimension or, as in the example, make it compact.

We aim of this project is to investigate a variety $X$ through such a group action. If we have a 1-parameter family of transformations $\{g_t\}$ of $X$, we can ask whether for fixed $x \in X$ the sequence $g_t(x)$ has a limit, the "final state" of $x$. If a limit exists for every point $x$, we can group the points according to where the limit lies. This is the core principle of Białynicki-Birula decomposition.

Our main goals for this project is to both apply and generalize this principle. We plan to

1. investigate generalizations of Białynicki-Birula decompositions from its usual setup of multiplicative group action, to the additive case.
2. understand the GIT and Chow quotients of several varieties with large symmetry groups.
3. apply the functorial version of Białynicki-Birula decomposition to very singular setups.

All these should yield new understanding of the geometry of spaces with large group action.