OPTIMAL REPRESENTATION OF DYNAMICAL SYSTEMS

The theory of dynamical systems is an important branch of mathematics related to the natural sciences and engineering. Inherently a dynamical system comprises of a *phase space* and an *evolution rule*. The phase space consists of all possible *world-states* (e.g., the locations and velocities of the planets of the solar system) whereas the evolution rule is a function which sends every world-state, representing the state of the system “now”, to the world-state representing the state of the system “one unit time later”. The weather, the production of white cells in the blood, the motion of billiard balls on a billiard table, the movement of gas molecules in a container, sugar dissolving in a cup of coffee, the stock market, the formation of traffic jams etc. etc. are all examples of dynamical systems. Successful modeling and analysis of systems within the scientific framework depends, as a first step, on a process of acquiring data from observations (e.g., measuring the pressure in a container containing gaz molecules). This process necessarily results with a *representation* of the system. It is of primary importance to determine how to acquire/represent a given system optimally from the point of view of precision, efficiency, reconstructability, storage constrains or other parameters depending on the exact circumstances. The goal of the project is to study optimal representation of dynamical systems arising in mathematical physics, stochastic compressed sensing, topological dynamics and symbolic dynamics. Appropriately, the project is divided into four parts.

To illustrate the contribution of the project consider a scientist observing a physical system (e.g., a complicated electrical circuit made out of many simple subcircuits) by measuring a quantity related to the system (e.g., measuring the electrical current at a specific location of the circuit) every second. This is a referred to as *delayed measurement* as it consists of measuring the same quantity at different (=”delayed”) times. It is of great practical value to understand how many delayed measurements (e.g, for how many seconds measure the current) one has to perform in order to gather sufficient information to simulate (= represent) the system on a computer. In the first part of the project, after formulating the problem in exact mathematical terms, our goal is to find the optimal (=minimal) number of delayed coordinates needed in terms of other available characteristics of the system, formally various *dimensions* (e.g., the number of simple subcircuits in the example above).

Let us further illustrate another part of the project which addresses *symbolic* representation of dynamical systems (i.e. by strings of 0’s and 1’s). This is always desirable as it is conducive to digital storage as implemented by modern computers. However for most dynamic systems precise symbolic representation (associating to each state of the system a *unique* string of 0’s and 1’s in a continuous way, respecting the dynamics) is impossible. So instead we study the question of finding a symbolic extension *onto* the system (reversing the arrow of the mapping above, i.e. associating to a closed set of strings of 0’s and 1’s states of the system in a continuous way, respecting the dynamics) whose *complexity*, is as close as possible to the complexity of the original system.

To summarize optimal representation of dynamical systems is an important subject in pure mathematics arising from practical yet indispensable considerations in various fields of natural sciences and engineering. By establishing new rigorous results in this area the project will contribute to the advancement of the mathematical discipline of dynamical systems as well as serve as a theoretical basis on which practitioners can rely in order to derive actual improved procedures.