Mathematical analysis of hydrodynamic models - nonlinearities, nonlocality, domain, scales

The aim of this project is to provide the mathematical analysis of nonlinear Partial Differential Equations (PDEs) and their solutions. In particular, we will concentrate on problems related to fluid mechanics, collective behaviour, interaction of objects/particles with fluid, phenomena observed in sciences and technology. It will give better understanding of some complex behaviours in considered systems. Well-posedness of the problem together with qualitative properties of solutions are our prime interest.

Many phenomena in nature, technology, sociology are described by models seeing it as a flow, therefore are presented as a basic systems of fluid mechanics. Although the systems of Navier-Stokes equations on a fixed domain are widely accepted for the description of flows of many liquids and gases, there is also a very wide class of phenomena for which this system is not sufficient to take into account more complex processes. Therefore, there is a need to construct and investigate models that take into account more complex nature of various phenomena.

In particular, in the comprehensive description of many phenomena the challenge is to take into account: collective behaviour and swarming of objects, microstructure associate with particles or objects interaction with fluid which are immersed in, non-Newtonian rheology of the fluid, changes of the shape and volume of the domain, heat effects, different scales of certain parameters which matters in the system and which values are dominant or negligible.

The above mentioned phenomena are a source of nonlocal effects, nonlinearities in the system, dependence on domain changes, and may change the character of the system respectively.

In particular, our goal will be to undertake research on the following issues:

- Analysis of the generalised hydrodynamic model of collective motion on time-dependent domain.
- Analysis of interaction of particles with non-Newtonian fluid on moving domain.
- Scale analysis of hydrodynamic models - low Mach number limit.

We expect to show:

- Existence of weak solutions and their behaviour for large times to the generalize Navier-Stokes-Vlasov equation on moving domain describing collective motion of particles or objects in non-Newtonian fluid.
- Existence of weak solutions and their behaviour for large times for generalized Navier-Stokes-Smoluchowsky equations describing interaction of particles with non-Newtonian fluid in so called bubbling regime. Periodicity of solutions in time at periodic movement of the domain.
- Rigorous mathematical justification for incompressible model describing evolution of supernovas being a low Mach number limit of compressible model.