

Imagine you are sitting on a chair, trying to move it just by swinging your legs. If you do it right, you will be able to force the chair to slide a bit without even touching the floor with your feet. In such a manner you, your chair, your legs and floor in your room form a controlled, discontinuous mechanical system. It is mechanical, because it obeys the laws of mechanics (as everything on Earth). It is controlled, because you decide about motion of your legs – you control it. Finally, it is discontinuous, because the friction force behaves in a specific way: if you gradually increase energy of swinging your legs, suddenly and unexpectedly, at some specific moment, the friction will be too weak to keep you in place and you will move some distance.

Now, the following question arises: how should you swing your legs in order to move the chair as fast as possible ? Should your swings be larger and take longer time, or be smaller and shorter ? This question belongs to a large family of Optimal Control Problems (OCPs). Scientists and engineers working in this field try to solve an extremely important issue: how to do something in a best possible manner, according to some criterion ? You can imagine how important this question is: you would surely like to have a car, which uses the as little fuel per 100km as possible, you want a fridge which does not waste energy or a computer which saves your time. All of these are special cases of the OCPs.

OCPs are not very easy in general, but if they concern discontinuous objects, they become even harder. So far, there are no really good ways of dealing with them. We can pretend that a discontinuous object can be described as if it was not discontinuous, but then our results will be either very inaccurate or very difficult to compute. Other possible methods are complex and time consuming.

We think that we have found a solution, which is going to make these problems easier. Here's how it will be done. We found a method using which your control (for example, the force and pace with which you swing your legs when sitting on the chair) can be described using just a few numbers. Moreover, our approach can take into account limitations (things like: you can't move your foot upper than your knee, you can't move legs as fast as you want etc.). And then, tuning a few numbers turns out to be a lot easier than adjusting the control throughout the whole time. Our initial hard problem is transformed into a much easier one. We expect that this method will solve many discontinuous OCPs (including, of course, you and your chair).

Last but not least, a man sitting on a chair and swinging his legs behaves in a very similar manner to a special, new kind of drive: a vibro-impact capsule. Inside it, there is a small oscillator (thing which moves forward and backward, just like your legs when swinging) and so the whole capsule is able to move (like your chair). Such capsule could be useful in many tasks. However, guys who work on this don't know how to optimally control it yet. Hopefully, they will be able to use our solution and create a device which will move in places where nothing else can.