MAJORITY COLORING OF GRAPHS

The aim of the project is to make substantial progress in a few open problems concerning graph coloring with a local propriety condition defined as follows. In every coloring of the vertices of a graph there may occur edges with differently colored ends (*good edges*) and monochromatic edges (*bad edges*). If at every vertex the number of incident bad edges does not exceed the number of good edges, then the coloring is called a *majority coloring* of a graph.

It is not hard to prove that every finite graph has a majority coloring using just two colors (it is sufficient to take a coloring minimizing the total number of bad edges). A guess that a similar statement holds for countable graphs is known as the *Unfriendly Partition Conjecture* and constitutes one of the greatest challenges in this area. Our aim is to attack this conjecture together with a few related problems including the *list, fractional* and *directed* versions (as well as all possible combined variants). For instance, the list version asks for the least value of k for which every countable graph is majority k-*choosable* (a color of each vertex is taken from arbitrarily imposed list of size k). The directed version concerns majority coloring of digraphs, in which the majority condition refers to the edges out-going from a vertex. The directed version of the Unfriendly Partition Conjecture states that every countable digraph is majority 3-colorable. In this case we don not even know if it holds for finite directed graphs. In the fractional version, in turn, one seeks the least possible fraction k/r such that there is an assignment of r-element subsets of a k-element set of colors to the vertices so that every coloring form these sets is a majority coloring. Proving that this fraction can be close to 2, or at least strictly smaller than 3, would be an important step towards the Unfriendly Partition Conjecture.

Our project has also a natural and more general goal: developing new techniques of tackling graph coloring problems with a defect of prescribed type and degree. One may expect that these techniques can be further used to derive approximate versions of diverse conjectures in graph coloring, where one allows for a badly colored part of the structure satisfying some specified restrictions. This may shed new light on difficulty of some long standing open problems in the area of graph coloring.