Abstract for the General Public

The objective of the project is to provide efficient algorithms solving problems such as scheduling problems of importance in aerospace: project scheduling, aircraft assembly or airport scheduling: gates, landing, parking. More generally, scheduling problems are *constraint satisfaction problems* where given a number of constraints imposed on the number of variables, one asks whether there is a global assignment to the variables such that all the constraints are satisfied. A constraint may enforce for instance that: *the task X is to be scheduled after task Y or after task Z.*

The research within the project employ a number of mathematical disciplines such as universal algebra, model theory, Ramsey theory and computational complexity. The research concern proving theorems which establish the computational complexity and provide algorithmic techniques for constraint satisfaction problems.

The importance of constraint satisfaction problems is of two origins. This class of problems forms a subclass of the class NP. It links our research to the most important open question in theoretical computer science, a Millennium Prize Problem of the Clay Mathematics Institute which asks whether NP equals P? In other words, it is asked whether problems such as Boolean satisfiability, graph coloring as well as many scheduling problems may be efficiently solved on computers. So far the humanity have not provided any efficient algorithms for these problems and expect that such algorithms do not exist. Though, no mathematical proof has been provided.

The solution of the P = NP? problem requires a better understanding of the nature of the problems in the class NP many of which, e.g., Boolean satisfiability or graph coloring, may be expressed as constraint satisfaction problems (CSPs) already when all constraints are modeled over domains with finitely many elements. Much understanding has been recently developed for these finite-domain CSPs taking advantage of so-called universal algebraic approach to the complexity of CSPs (i.e., an approach based on mathematical discipline — universal algebra). In particular, we learned that there exists a dichotomy, i.e., all finite-domain CSP are either NP-complete (i.e., practically unsolvable by known algorithm) or in P (i.e., "efficiently" solvable).

The problem with finite-domain CSP is that they cannot express many practical problems such as scheduling or problems in temporal and spatial reasoning. Fortunately, we have recently learned that universal algebra this time along with model theory (a discipline of mathematical logic) and Ramsey theory is of use to study the complexity of a class of infinite-domain CSPs, which can express scheduling as well as problems of vital importance in temporal and spatial reasoning. Though, the full understanding of this class of infinite-domain CSPs is still far away. In particular, it is not clear which algorithms could place some of the considered problems in P. It is exactly where the current project comes in. The main goal of the project is to use universal algebra, model theory and Ramsey theory to provide the appropriate algorithmic tools and their understanding.

Effects of the project will be new efficient algorithms and the understanding of the computational complexity of constraint satisfaction problems over infinite domains. Defining scheduling problems as CSPs is not only beneficial from a theoretical perspective. On the practical side, many common tools are designed to incorporate the formalism of CSPs (take for example the algebraical scheduling language offered in IBM ILOG CP Optimizer, which is a component of IBM ILOG CPLEX Optimization Studio).