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The project focuses on applications of set-theoretic methods like infinitary combinatorics, set-theoretic forcing, uses of additional set theoretic axioms, etc., in topological aspects of some topics of classical abstract functional analysis such as isomorphic theory of Banach spaces, geometry of Banach spaces, algebras of operators acting on Banach spaces, operator algebras (C*-algebras), spaces of Radon measures.

The source of unifying topological motivations is the set C(K) of all real or complex valued function on a Hausdorff compact space K. It carries the structure of a Banach space with the supremum norm or the structure of a commutative C*-algebra or can be considered as a topological space $C_w(K)$ with the weak topology induced by the Banach space structure or the topology of pointwise convergence $C_p(K)$.

To get the full picture it is natural and useful to consider not only compact Hausdorff Ks but their subspaces X (this is exactly the class of Tychonoff spaces) and the corresponding space $C_p(X)$ as well as locally compact Ks and the Banach space $C_0(K)$ of continuous functions on K vanishing at infinity. The examples of main objects involved are the Čech-Stone compactification of the naturals $\beta \mathbb{N}$, the Banach space of bounded sequences ℓ_{∞} , the space of bounded operators on the Hilbert space $\mathcal{B}(\ell_2)$ and their substructures and quotients; the dual balls B_{E^*} with the weak^{*} topology, for subspaces E of ℓ_{∞} or subalgebras E of $B(\ell_2)$, their subspaces and continuous images and spaces of continuous functions defined on them with the different topologies mentioned above; the spaces of measures on some of the spaces mentioned.

The project concerns the objects (mainly function spaces) which are important both in topology and functional analysis. The topics of the project attract the attention of mathematicians from various research centers in Australia, Austria, Brazil, Canada, the Czech Republic, France, Greece, Great Britain, Mexico, the Netherlands, Poland, Russia Spain, USA, and other countries. Research in this area require combination of methods from different branches of mathematics - general and infinite-dimensional topology, Banach spaces, C*-algebras, set theory, measure theory, descriptive set-theory. Numerous recent results in the theory of Banach spaces and operator algebras, obtained by us and other mathematicians, demonstrate that the application of methods from combinatorics and set theory can be very fruitful. In some cases, the use of techniques from set theory is absolutely necessary, the answers to some well-known problems form analysis depend on some additional set theoretical axioms. What's more, we can deal with a rather surprising situation: assuming an additional axiom of set theory, a given problem may have an affirmative solution, and assuming another one, negative; in other words, it cannot be resolved on the basis of the standard set theory.

This interaction between different fields of mathematics makes this area of research particularly interesting and stimulates the development of new methods and techniques in all involved fields.