"Concentration of measure and functional inequalities" Description for the general public

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A classical observation lying at the foundations of the modern probability theory says that if we run a random experiment sufficiently many times, then the resulting average outcome is very close to the expected outcome of a single experiment. This observation was originally formalized in terms of various laws of large numbers, which are qualitative in nature, i.e. they tell us that that with increasing number of trials we get closer to the expected value but do not tell us how close we get. Being able to estimate this error is a crucial task from the point of view of many applications and a large body of research is directed toward providing good estimates of this type.

It turns out that in many situations one is able to show that the bound on the probability of the error being big decays very rapidly (i.e. at least exponentially). This is a desired situation since it tells us we do not need many trials to get precise estimates with high probability. The prevalence and strength of such bounds is a fascinating matter, being referred to as the concentration of measure phenomenon, which in its basic form has been neatly phrased by Michael Talagrand:

> a random variable that depends (in a "smooth" way) on the influence of many independent variables (but not too much on any of them) is essentially constant

In practice however, the above assumptions are often too restrictive and one has to deal with models of random variables that are highly dependent or the class of functions of interest is much wider. Such problems occur when analyzing models that appear naturally in e.g. statistical mechanics, social networks, machine learning or biology to name only a few areas.

The objective of this project is to investigate concentration inequalities and the interplay between various approaches that lead to them. In particular, we will be interested in the following two contexts:

1. Concentration for non-Lipschitz functions in dependent random variables.

2. Functional inequalities.

The first goal of the project is to study the concentration of measure in the probabilistic and combinatorial setting. We will be analyzing specific, mostly discrete, models of dependent random variables that arise naturally in applications and deduce their concentration properties. We will be also interested in formulating some sufficient conditions for specific types of concentration (e.g. concentration for convex functions or polynomials) to hold and show they are satisfied in some important situations, improving previously known bounds.

The second goal of the project is to investigate the abstract analytical setting of functional inequalities. These can be often seen as a bridge between the theory of convergence to stationarity of Markov processes and concentration properties of limiting distributions. In particular, we will be analyzing the dependence between various types of functional inequalities, deriving their characterization and investigating what concentration properties they yield.

As a result of the conducted research, new concentration inequalities will be obtained and new connections between various ways of approaching these inequalities will be found and proved which will lead to the better understanding of many important models of applied and pure mathematics.