Boundaries of Helly groups

abstract for the general public

The notion of a group is essential in mathematics. It serves as a device to express symmetries of objects.

Geometric group theory studies connections between groups and spaces (that satisfy some geometric conditions), upon which they act (expressing their symmetries). This way we get a link between two areas of mathematics – geometry and algebra, which allows us for a better understanding of groups by studying spaces and *vice versa*. The research field originated in the 1980s and was inspired by combinatorial group theory, algebraic topology and differential geometry.

Our project follows the trend of studying groups that act on non-positively curved spaces. The notion of non-positive curvature originates from differential geometry; it is connected to the fact, that the balls in non-positively curved spaces are convex.

In our project, we plan to focus on groups that act on Helly graphs or on injective spaces, or, in greater generality, on so called bicombable spaces (i.e. spaces, for that one can assign to each pair of points a curve of minimal length among the ones that join them, a so called geodesic, in a way continuously depending on the endpoints and satisfying some additional properties); all of these three classes can be treated as examples of classes of non-positively curved spaces. It is a relatively new research direction in geometric group theory, yet, interestingly, the notion of an injective space and of Helly graph appeared in the development of mathematics multiple times, in its different areas, being defined in several ways, all of which later turned out to be equivalent. The fact, that a space is injective, causes it to have a bicombing, which makes it similar, in some sense, to so called CAT(0) spaces, which form another class of non-positively curved spaces; in a CAT(0) space each triangle with sides being geodesics is not fatter than the triangle on the plane with the same lengths of sides.

This research project focuses on studying boundaries of groups (at infinity). Simplifying, they are created in the following way. According to the main idea of geometric group theory, instead of a group, we can consider a space on that this group acts *nicely*. We pick a point in this space and look at the set of points in a fixed distance from that point, we can think of it as of a "boundary" of the set of all points in distance not greater than the fixed one. One may then observe that, when increasing the distance, the picture we can see starts to increasingly resemble some very regular, yet complicated, space – a boundary of the group.

The goal of the project is to begin systematic research of boundaries of groups that act on Helly graphs, injective spaces or bicombable spaces. This subject is new, the notions it uses have not been defined nor studied in great detail. Better understanding of boundaries should allow for a better understanding of known examples of groups and creating new ones, having potentially some exotic properties.