POLS DynComP project
Abstract for the general public

Dynamical systems are models of all kind of phenomena evolving in time. Their analysis helps us to understand and predict the future of processes that are described via equations and functions. Thereby, the best possible situation we could imagine is to completely classify all possible outcomes of processes with arbitrary accuracy. Indeed, in a sense, this is the quintessentially goal of the theory of dynamical systems, namely, to classify all possible dynamical systems via their long-term behavior in a meaningful and absolute fashion.

Of course, in its total generality, this is a Herculean task and depending on the concrete level of resolution of our desired classification, even an impossible task. It follows that in order to proceed in this endeavor we have to restrict ourselves to more concrete situation, i.e., we have to consider particular families of dynamical systems. Moreover, we have to come up with feasible schemes and suitable defined concepts to establish meaningful and doable classifications.

A very common scheme and successful approach to obtain useful classifications of dynamical systems is the following: first, we have to define when we want to consider two dynamical systems to be the same (in this case, we will also say that the two system are *isomorphic*). Here, two very prominent concepts are topological conjugacy and measure-theoretic isomorphism. Second, we have to come up with convenient dynamical notions which do not change for systems which are isomorphic. Such notions are called *dynamical invariants* and they usually reflect different kinds of dynamical behaviour and complexity of a system. In the best case possible, these invariants can be expressed as a single number. Note that in case we have a dynamical invariant which differs for two dynamical systems, then these two systems cannot be isomorphic.

Now, one of the most prominent dynamical invariants is the notion of *entropy*. It measures how much disorder is present in a system, by quantifying the exponential growth rate for the number of initial states that can be separated within a certain accuracy while time passes and accuracy increases. As it turns out, entropy is especially useful for classifying systems that exhibited a lot of complexity (sometimes referred to as chaotic systems). However, this project is devoted to explore and study dynamical invariants for systems showing only low-complexity behavior (zero entropy). For these systems one needs to develop new invariants.

The class of dynamical systems in the low-complexity regime is very rich. For example, it contains systems which are intimately linked to so-called *quasicrystals*. From the perspective of physics, the notion of a quasicrystal refers to a structure without translational symmetries but with a certain long-range order which results in a (pure) point diffraction pattern (a phenomena which is classically only associated with periodic structures like lattices). Moreover, substitutive systems, i.e. systems which are defined by finitely many local rules (similar to famous Game of Life of Conway), show usually low-complexity behavior and play a central role in this project. Accordingly, to understand and classify these kind of systems is a very desirable task and we hope that we can contribute to this endeavor.

Finally, it is very natural to ask whether there exist general strategies to find interesting and useful dynamical invariants. As it turns out one fruitful strategy is to investigate suitable defined *pseudometrics*. Usually, these pseudometrics measure in an averaged sense how alike different initial states of a dynamical system will behave in the future. One aim of our project will be to demonstrate that these dynamical invariants induced by pseudometrics are suitable to study low-complexity systems and to take a step towards providing fundamental classifications-schemes for these systems.