

Euclid's *Elements* includes basic techniques of ancient Greek mathematics. With regard to geometry, these are: the properties of triangles and circles, the theory of equal figures, the theory of proportion, the similarity of figures, and the method of exhaustion. Over the course of the history of mathematics, the arithmetic of real numbers replaced the theory of proportions, and the Riemann integral replaced the exhaustion method. Arguments based on the properties of triangles and circles in their original forms are the part of elementary and high-school geometry curriculum. The similarity of figures is also the part of school curriculum, however, while the propositions of this theory are more or less like the ancient ones, their proof technique is modern, namely, it involves the arithmetic of real numbers.

Another way of referencing an antique theory is its interpretation in a novel theory, resulting in the original propositions attaining new meanings. The history of the Pythagorean theorem provides a model example. In the *Elements*, it was about squares constructed on the sides of a right-angle triangle. In *La Géométrie* (1637), Descartes gave it the algebraic form $a^2 + b^2 = c^2$, where a, b, c are line segments, while a^2, b^2, c^2 are the products of line segments defined in a special way. In modern geometry, *La Géométrie* formula represents the Pythagorean theorem, although a, b, c stand for real numbers, that is, the lengths of the sides of the triangle. A coding of the properties of a triangle in trigonometric identities provides yet another interpretation of the ancient theory. This way of dealing with triangles is documented in Robert Simons' edition of the *Elements* dated 1756. The theory of similar triangles included in Hilbert's *Grundlagen der Geometrie* (1899) is another interpretation of the *Elements*.

Until the 17th century, Euclid's *Elements* were treated as a source of mathematical knowledge. In the following centuries, its status has changed from a mathematical treaty to a mathematical textbook. In the 20th century, it became the subject of philosophical investigations, especially in the methodology of mathematics.

There are essentially two ways of interpreting the *Elements*. The first originates in Hilbert's *Grundlagen der Geometrie*. It consists of a reconstruction of the logical structure of the treaty. Euclid's propositions are derived in a system that contains Euclid's axioms as well as some new ones. This interpretation pays no attention to the role of diagrams. Hilbert's project is developed further by Hartshorne in his book (Hartshorne 2000). The second trend respects the fact that in many proofs, Euclid refers to diagrams. This kind of interpretation involves assigning a special role to diagrams in mathematical argumentations. This project was initiated by K. Manders' article *The Euclidean Diagram* (2008), and then developed into a formal system in a series of papers by J. Mumma and J. Avigad, e.g. *A Formal System for Euclid's Elements* (2009), *Proofs, Pictures, and Euclid* (2012) or *The Eu approach to formalizing Euclid* (2019).

As part of our project, we will present an interpretation of the *Elements* that reconstructs both the theses of Euclid's propositions as well as his proof technique. We do this on the basis of book VI of the *Elements*. The theory of proportions adopted by Hilbert, and later by Hartshorne, is not able to reconstruct Euclid's theorems in which the figures are terms in proportion, e.g. VI.1, 20, 31. In turn, authors who underline the role of diagrams cannot explain the comparison of figures in terms of greater–lesser, especially when one figure is not contained in the other.

In our proposed reconstruction, we apply the area method, as developed in the book Chou, Gao, Zhang, *Machine Proofs in Geometry* (1994). It allows us to reconstruct both Euclid's theses and his proof technique, which in the case of book VI means the theory of proportion. Moreover, while in Hilbert's tradition, the processing of information is understood as the logic of sentences, in the second tradition, it is a way of processing diagrams. In the area method, as well as our interpretation of Euclid, the processing of information is interpreted as symbolic calculation.