Macroscopic description of some physical models using methods of probability theory
(a brief general description)

The present project is devoted to the mathematical investigation of some physical phenomena in which an important role is played by methods that are rooted in the theory of probability and stochastic processes. In particular, the problems we plan to study are motivated by the phenomena connected to the theory of heat conduction and propagation of quantum and classical waves in complicated media. Due to the high complexity of the systems we have in mind (crystals and fluids made of molecules, or atoms, or a motion of an electron in a random environment) it is quite natural to describe their evolution using models that involve stochastic processes, or random fields. Frequently the interest in such models stems from the fact that one wishes to obtain a macroscopic description of the behaviour of the system, while the model itself is formulated at the microscopic scales. To achieve this goal one may avail oneself of the techniques of the theory of probability and stochastic processes.

Our present project consists of two parts. In the first one, we propose to investigate the problem of heat transport in a one dimensional medium. In this case the heat conductor is modelled by a chain of harmonic oscillators with a stochastic perturbation of their momenta. We would like to investigate finite and infinite open systems, i.e. such that are in contact with their surroundings either via some thermostats (that maintain fixed temperatures), or external forces. We wish to show that the macroscopic energy density for such a system can be described by a fractional diffusion equation with some boundary conditions. In addition, in this part of our project we plan to investigate the anomalous scaled limits of kinetic equations with boundary conditions and degenerating scattering kernels. Such equations describe the resolution of the density of phonons, i.e. theoretical particles appearing in the mezoscopic description of a vibrating chain, in both spatial and frequency coordinates

In the second part of the project, we plan to investigate the topic of partial differential, or integro-differential equations with random coefficients. The first problem we wish to undertake deals with the Schrödinger equation with a random potential. This equation is important both in quantum mechanics and in the propagation of classical waves in disorganized media, where it appears in the so called paraxial approximation of the wave equation. It is expected that in the macroscopic scaling the transport of the energy of the wave can be described by a diffusion equation (the so called diffusion approximation). The author of the grant project has already published several papers concerning this phenomenon in the case of weak potentials (the so called weak coupling regime). We plan to investigate the case of strong potentials that are δ-correlated in time - the so called Itô-Schrödinger model.

The second problem we are going to undertake in the current project is the problem of homogenization for solutions of integro-differential equations with random coefficients. It concerns the averaged description of the solutions when passing to the macroscopic scales. It is expected that then their limiting behavior is described by solutions of equations with constant coefficients. Since the aforementioned procedure removes inhomogeneity in the description of the environment, it is called homogenization. The author has published several dozens of papers concerning the phenomenon of homogenization for solutions of partial differential equations with random coefficients. In the present project we plan to use probabilistic methods to investigate the problem of homogenization for equations of this type.