

The project concerns mathematical analysis of certain variational methods used to perform tasks in image and data processing, such as denoising, enhancement or decomposition. The common idea of investigated methods is to decrease in controlled way certain quantity E associated with the image, such as the total variation (TV). For example, in the case of monochrome bitmap graphics, the (anisotropic) total variation can be calculated by summing the absolute value of difference between brightness of all adjacent pixels. In simplification, this can be considered as a measure of noise of the image. *To decrease E in controlled way* means that a new image is selected that minimizes E among all possible images which are, in a sense, not too different from the original image. Alternatively, a so-called steepest descent flow of E can be used. The class of quantities E considered is chosen so that the corresponding decreasing procedure removes salt-and-pepper type noise, but does not blur out definite sharp contours in the image.

The question to be investigated in the first part of this project is how do certain mathematical properties of the image change after applying the procedure. For example, we ask whether new sharp contours can appear in the image. We also investigate the behavior of so-called Sobolev seminorms, which can be thought of as measures of local variance of the image.

An important novel feature of the project is that we allow for multichannel images/data, such as RGB images, in which three numbers describing intensity of base colors are assigned to every pixel. The second part of the project concerns data which furthermore satisfy a nonlinear constraint, such as the color component of an RGB image, which lies in the sphere. Other examples of such data are orientations of objects such as cameras or airplanes in space (which correspond to elements of so-called Lie groups such as $SO(3)$ or $SE(3)$) or diffusion tensor images (in which case a 3×3 symmetric positive-definite matrix is assigned to each pixel). We investigate mathematical well-posedness of the steepest descent flow of total variation of such data. That is, we investigate existence and uniqueness of solutions to differential equations governing such flow. This kind of questions is fundamental for understanding qualitative behavior of the flow, as well as for understanding the properties of numerical schemes used to compute the flow.