Abstract for the general public

Symmetry is associated with beauty and harmony. In mathematics, the analysis of symmetries is a basic method of investigating mathematical objects, e.g. geometrical figures, and more symmetric objects are easier to describe. The intuition behind symmetry is rigorously encoded in the concept of an action of a group on a space. This abstract approach enables mathematical modelling of physical objects such as crystals, and the search of good symmetries leads to the theory of principal bundles, which constitute the mathematical language of gauge field theories in physics.

Another fundamental method of the analysis of mathematical objects is their classification. In topology, to distinguish different principal bundles, one introduces the classifying space, which is unique for a given symmetry group. It turns out, that a similar classification is possible for a different kind of symmetry, i.e. proper actions of groups on spaces. Proper actions and their classifying spaces are related to the celebrated Baum–Connes conjecture.

Symmetries can also occur in entirely different situations. In quantum mechanics, we cannot see the particles, but we can measure their momenta or positions. From mathematical point of view, these measurements are represented by eigenvalues of some noncommutative operators. We can still talk about the symmetry of this system using the concept of an action of a quantum group on an algebra of operators. This is the starting point of noncommutative topology, which generalizes the classical topology and studies noncommutative algebras of operators, e.g. C*-algebras.

The aim of the project is to find noncommutative counterparts of classifying spaces for principal bundles and proper actions. After the recent generalization of the notion of a principal bundle to the noncommutative world, the classification of such bundles is a natural next step. Classifying spaces are fundamental in topology, hence, we believe that these new objects will play a similar role. Possible applications include new formulations of the Atiyah–Segal completion theorem for C*-algebras and the Baum–Conness conjecture for quantum groups. At the same time, our investigations will shed a new light on various generalizations of free and proper actions to the context of quantum-group actions on C*-algebras.