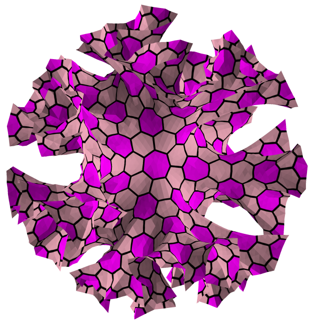

Non-Euclidean geometries were discovered by mathematicians wondering about the nature of parallel lines. Euclid has shown that all known geometry is a consequence of five simple axioms. However, he was not completely satisfied with his fifth axiom, which stipulated that lines which do not cross stay in a fixed distance. This axiom was more complex to state than the other four, and thus Euclid thought that it was a consequence of them, but he was unable to prove it. In the 19th century hyperbolic geometry was discovered, where the fifth axiom does not hold – in hyperbolic geometry, the lines which do not cross "diverge". Hyperbolic geometry can be seen as the geometry of a surface with constant negative curvature (shown in the picture). A more common non-Euclidean geometry is the geometry of a surface of constant positive curvature (sphere); the problems caused by the surface of Earth not being flat (and thus non-Euclidean) are evident, e.g., in cartography and navigation. There are more possible geometries in higher dimensions.While originally the research in non-Euclidean geometry was very abstract and even considered insane (the fifth axiom clearly holds in our world, so why care), eventually it led to the development of differential geometry, currently widely used in many fields, from physics (general relativity theory) to data analysis and engineering.

In our project, we aim to research, among others, the following areas:

Visualization of Thurston geometries. One of the most important recent results in mathematics is the proof of Thurston's geometrization conjecture. This conjecture states that every three-dimensional space can be subdivided into parts, each of which has Euclidean, hyperbolic, spherical, or one of five other specific geometries. One of consequences of this conjecture is the Poincaré conjecture, for solving which a prize of 1 million USD was offered; the author of the proof, G. Perelman, famously did not accept the prize. One of aims of our project is to visualize these geometries in an attractive way: the user will be able to virtually enter a world based on one of them. No such visualization exists yet, and thus it will be valuable for mathematicians working in low-dimensional topology, physicists, mathematical artists, video game developers, and data scientists.

Algorithmic aspects of hyperbolic graphs. Hyperbolic geometry is closely related to the subfields of theoretical computer science which are strong in Poland, such as fixed parameter complexity, or the theory of automata on infinite trees. It is known that tree-like graphs have very good algorithmic properties; the tree-likeness is usually measured by *tree-width*. Hyperbolic graphs are also similar to trees, but this similarity is measured in other ways. We aim to characterize algorithmic problems which can be efficiently solved on such graphs.

Embedding data into non-Euclidean spaces. A hyperbolic space has structure similar to an exponentially expanding tree. Try to draw a full binary tree of height 10 of a piece of paper – two branches, each of them having two subbranches (every branch and subbranch of the same length), and so on, 10 times. You will probably not succeed, or the 10th level branches will be very close together. This problem does not occur in the hyperbolic plane – since the parallel lines "diverge", free space appears between them, making the big tree fit. This property has recently found application in data analysis and visualization – whenever we want to visualize or model data with tree-like structure. For example, social networks such as Facebook or Twitter have a similar structure – posts written by the most popular people are read in the whole world, while those written by less popular people are read by their local acquaintances, or people with similar interests. Our project aims to study embeddings in other non-Euclidean spaces, and study the possibility of using such embeddings to detect important nodes in social networks.