The project is divided into three parts. In the first we deal with, so called, limit theorems for random processes. To illustrate what it actually is, let us consider the following game. We fix a favorite currency, say dollars, and toss a coin. If the result is a tail, we win one dollar, and if the result is head, we lost one dollar. Denote by S_n amount of money we have after n tosses. Intuitively we expect that after a large number of tosses our average gain in one game is close to zero, i.e. $\frac{S_n}{n} \sim 0$. This is called the law of large numbers. However, we would like to know more: what is a probability that our total gain S_n is in some interval $[-m, m]$ for some $m > 0$? This can be approximated using so called normal distribution. This limit theorem is called the central limit theorem. Finally, what is the maximal and minimal gain we can have after a large number of tosses? Clearly it should depend on the number of games and may not be just equal to it, since it is not reasonable to expect that there will be no head for the whole time. One can prove that $-C\sigma\sqrt{2n \log \log n} \leq S_n \leq C\sigma\sqrt{2n \log \log n}$ almost surely for some $\sigma > 0$, any choice of the number $C > 1$ and for sufficiently large n. This estimate is sharp. The last limit theorem is called if the law of the iterated logarithm, because of the strange sequence involved containing iteration of logarithm $\log \log n$.

In the project we would like to investigate games arising from random walks on the interval. Take any continuous function φ on the interval such that when picking randomly a point in the interval the expected value of φ at chosen point is equal to zero. Then consider some rule of moving on the interval. For example, let as choose some point $x \in (0, 1)$. If this point is on the left-hand side of the number $3/4$, then go to the point which is two thirds times closer to zero than the initial x. If the point is on the right-hand side, then go to the point which is two times further from one than the initial one. The second rule will be symmetric with respect to the first (actually, we consider much more complicated rules). Now we toss a coin again and move with respect to the first rule if the result is tail. When it comes down a head go with respect to the second. This is called a random walk on the interval. Its result is a random sequence of points $(x_1, x_2, \ldots)$ of points visited during this walk. Our gain is given by the numbers $S_n = \varphi(x_1) + \varphi(x_2) + \ldots + \varphi(x_n)$. The question is if the limit theorems hold.

The second part of the project is also devoted to random walks, this time on the circle. Fix $\alpha \in [0, 2\pi)$ non comeasurable with 2π and a assign to each point of the circle an asymmetric coin, i. e. such that the probability of coming down a tail is not necessarily equal to the probability of coming down a head. This assignment cannot be arbitrary. The probability of coming down a tail should depend in a very regular way on the point of the circle (at least continuous). Then consider the random walk: if you are at a point x, take the coin assigned to this point and go to the point which is at distance α in the counterclockwise direction. If it comes down a head, go to the point which is at the same distance but in the opposite, hence clockwise, direction. If you fix an arc on the circle, you may ask what is the probability that you are in the interval after a large number of steps. Is it stabilizing when the numbers of steps is increasing?

The third part is devoted to ergodic theory. To illustrate what this theory is, imagine a closed box containing some gas which is somehow mixed. This gas actually consists of a large number of tiny particles moving all the time. One can fix one particle and observe its trajectory. What its statistical behavior is? How much time does it spend in some fixed part of the box? What does it depend on? Ergodic theory deals with questions of this type. We consider certain examples of systems, in which the phase space is the circle. The circle is the simplest example of a torus, which is a phase space for problems coming from celestial mechanics.