Lipschitz geometry of metric spaces, its operator theory, and harmonic analysis
–abstract,
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The distance between two points is certainly a non-negative number that does not depend on whether we measure it from the initial point to the end-point, or the other way around; every point has distance zero to itself; moreover we may only increase the distance if we travel from one point or another via some other point. In Mathematics, sets endowed with the notion of a distance that meets the above-mentioned properties are called *metric spaces*. Metric spaces are ubiquitous in Mathematics because they allow for convenient unification of methods from various branches of Mathematics. Indeed, we often consider metric spaces of functions, sets, or more general objects. Distances other than the Euclidean one, that is the ordinary distance we use in our daily life, are often employed in Statistics or Data Analysis.

Metric spaces, as defined, do not allow for addition or scaling of vectors–operations that we may freely perform on the plane or in the three-dimensional space. In Mathematics, *normed spaces* are a suitable class of metric space, where these operations are available. These structures are rich enough for the development of the classical differential and integral calculus.

The primary aim of the present project is to study distance-preserving embeddings of general metric spaces (without any extra structure) into normed spaces. The resulting construction is called the *free Banach space over a metric space* and is an object of advanced study of a rapidly developing branch of contemporary Mathematical analysis.

One of our objectives that we wish to purse is to employ continuous linear maps between free Banach spaces, that is maps that continuously preserve the addition and scaling (we call them linear operators) for the sake of studying the fine structure of free Banach spaces, and that in order to recover information of the ambient metric spaces. If we consider the process of assigning the free Banach space to a metric space going one step up, in our project we propose to actually go even one step further.

The philosophy of using linear operator may very likely open the door for the development of another area of Mathematics, namely Harmonic analysis (that has its roots in signal processing) in the context of metric spaces that carry an extra structure of a semigroup, that is, they are endowed with an associative operation. Harmonic analysis is well-developed within the class of the so-called locally compact groups, but beyond that class not much is known.

Furthermore, we wish to use this very apparatus to study coarse geometry of metric spaces. Roughly speaking, the obviously very different sets of integers and of real numbers look similarly when we look at them from a large distance, indeed, the integral points seem to merge into a straight line. In that case, we say that these sets are coarsely equivalent. One of the approaches used in coarse geometry is to use certain operator algebras of operators on the space of square-summable functions on the underlying metric space. We wish to reconcile this approach with using operators on free Banach spaces over metric spaces.

As for the effects of a successful completion of our project, we recognize the development of important areas of Mathematics such as Harmonic analysis or Operator theory in the context of naturally occurring classes of metric objects for which these are currently unavailable. Furthermore, another effect would be introduction research trends becoming part of the mainstream of contemporary Analysis on the Polish soil.