CHAIN - Generic chaining approach to the regularity of stochastic processes

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Stochastic processes are considered whenever the mathematical analysis of a random phenomenon depends on time. The process is a family of random variables indexed by a fixed set. In many problems that are relevant to probability, not only are important processes indexed by time interval but above all processes indexed by arbitrary sets. For example, the norm of a random vector in a separable Banach space can be treated as a supremum of the evaluation-type stochastic process indexed by all norming linear functionals. A sample path is a single outcome of a stochastic process, so it is formed by taking a single possible value of each random variable of the stochastic process. The main objective of the project CHAIN is the regularity of paths in various open questions that are considered in the probability theory. The project is a continuation of previous PI investigations within the project BOUNDS which eventually resulted in the resolution of several intriguing questions that can now be further developed. The key tool on which the project will focus is the construction of appropriate generic chainings, whose main idea is to approximate index sets of high complexity through suitable sequences of finite subsets. In general, it is easier to prove upper bounds using the chain technique and it is much harder to prove the corresponding lower bounds. For example, the fundamental question for the general theory of processes is the characterization of expectation of norms of random vectors, especially in the case of vectors with dependent entries such as vectors of log-concave one-unconditional distribution. The crucial thing to make this analysis possible is to show the Sudakov minoration, a property which combines a geometric information about the separation of points in a set of indexes with a lower bound on the expected value of the supremum of processes indexed by such a set. Only recently has PI found the right approach to proving the property for a suitably broad class of random vectors. Together with the techniques invented by prof. M. Talagrand this always leads to the characterization of the expected values of some random vectors, at least under some regularity conditions. On the other hand, the interpolation method invented by prof. R. Van-Handel shows that when establishing lower bounds, some assumptions required in Talagrand's approach can be weakened.. Note also that prof. M. Talagrand has been writing a new version of his seminal monograph on upper and lower bounds for stochastic processes, whose draft version was the inspiration for several specific objectives of CHAIN. All these arguments mean that the project should succeed.