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KLEIN COVERINGS OF COMPLEX CURVES

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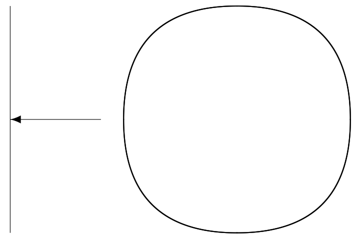
Geometric ideas. Our basic objects of study are smooth curves (no crossing, no cusps). Imagine a closed segment and a circle of the same size on the right (as in the picture below).

The map from the circle to the segment has a nice property. Every point in the interval is an image of two points of the circle (apart from the end points). Hence, this map is an example of a double covering ramified in 2 points – the ends of the segment. Can you draw a double covering of a circle?



Our project deals with complex curves (the complex numbers are such that $\sqrt{-1}$ makes sense). Roughly speaking these are special kind of real surfaces called Riemann surfaces. If we consider the sphere (which is called the Riemann sphere) instead of an interval, then any surface which is its double covering (with at least 6 ramification points) is called a hyperelliptic curve. In the project we would like to study Klein coverings of hyperelliptic curves. A Klein covering is an unramified covering of degree 4 with the monodromy group isomorphic to the Klein four-group).

More precise statements. Coverings of curves allow us to compare properties of the curves upstairs and downstairs. In curve theory we know a lot about double coverings. The topic of the project are Klein coverings which are natural generalisations of double coverings. Notice that $4 = 2 \cdot 2$, hence one can treat a Klein covering as a composition of two double coverings. From this point of view, our investigation is new, yet very natural.

In papers (joint with Angela Ortega from Humboldt University in Berlin) entitled *Hyperelliptic curves on* $(1, 4)$ *polarised abelian surfaces* and *Klein coverings of genus 2 curves* we have shown some results concerning Klein coverings of genus 2 curves, especially the injectivity of the Prym map. In the project, we would like to generalise our results in the following (mathematically more precise) way.

- a generalisation of the construction to higher genera, at least in the case when a base curve is hyperelliptic.
- a description of hyperelliptic curves with involutions that have fixed points
- a construction of explicit models: we would like to write equations of curves and period matrices of Jacobians that appear in the construction for genus 2.
- a compactification of the construction by considering singular genus 2 curves and their Prym maps
- a generalisation of the construction to $\mathbb{Z}_3 \times \mathbb{Z}_3$ coverings and understand the Prym varieties in this case.