Selected interpolation nodes in polynomial approximation

Polynomial interpolation gives one of the simplest ways to approximation of data or functions. For given pairs $(x_1, y_1), ..., (x_n, y_n)$ we can find an interpolation polynomial p such that $p(x_1) = y_1, ..., p(x_n) = y_n$ and we say that p interpolates the values $y_1, ..., y_n$ at the nodes $x_1, ..., x_n$. We expect that p is of lowest possible degree. If $y_1 = f(x_1), ..., y_n = f(x_n)$ for a function f then p is called an interpolation polynomial of f. By means of these polynomials we can approximate complicated functions by polynomials that are relatively simple.

This approximation is possible if we have "good" nodes. A choice of appropriate nodes is a complicated problem and depends on a set where approximation is needed. By some theoretical results, we know that there exist many good nodes. However, we can find them efficiently only for a few specific sets like a segment or a disc. Therefore, we are looking for other concrete examples of sets where we can find exact good nodes.

It seems that the simplest way of construction of good nodes for a given set is calculating them numerically. Unfortunately, this often requires us to solve a very hard optimization problem. This is a reason for which we are searching other new algorithms.

The project deals with problems that are currently intensively investigated. Finding good interpolation nodes is a key problem for numerical methods of solving partial differential equations. Therefore, this topic has an impact on a development of various fields of science. Some results connected with this subject found some important applications e.g., in Magnetic Particle Imaging (MPI) in the clinical medicine in diagnostic tests. A progress in interpolation theory has important consequences and applications in image reconstruction and comparison or data processing and transfer which is useful e.g., in clinical medicine, smartphone technology etc.