The objective of the project is to construct the theory of combinatorial dynamical systems together with algorithms and software for the analysis of sampled dynamical systems. To explain our motivation we need to recall what is the theory of dynamical systems and what are its deficiences in relation to the current needs of sciences and technology.

The discovery of infinitesimal calculus by Leibniz and Newton in the second half of 17th century enabled the formulation of problems in science in the form of differential equations. Although few such equation can be solved analytically, numerical methods developed in the 18th and 19th century let scientists construct sufficiently good approximate solutions. The construction of digital computers in the second half of the 20th century has significantly increased the range of applications of numerically solvable differential equations. Unfortunately, numerical methods can only provide solutions over finite time intervals. In consequence, the approximations often do not suffice to predict the asymptotic behavior of solutions that is the behavior as times goes to infinity. The problem became apparent at the end of 19th century when scientists tried to prove the stability of our Solar system. The stability problem belongs to a broader class of problems consisting in the classification of invariant sets and, particularly, attractors without the knowledge of the exact or approximate formulas for solutions. In such problems the unknown solutions become the object of study of the theory of dynamical systems. At the end of the 19th century French scientist Henri Poincaré invented topology as a new tool to attack problems in the theory of dynamical systems. In the second half of the 20th century several topological tools have been proposed to study and classify the asymptotic dynamics, in particular the Conley index based on Ważewski principle. The beginning of 21st century brought new challenges. Modern technology facilitates acquiring terabytes of data from experiments, observations or simulations. The problem known as Big Data is how to extract useful information from huge amounts of data. In geometric terms such data forms a cloud of points in a high dimensional space. The cloud approximates a closed hyper-surface in the space and the topology of this hyper-surface provides useful clues what information is hidden in the data. The reconstruction of the hyper-surface from data in the form of an approximating polyhedron is the subject of intensive research in recent years. For many dynamical problems in sciences, particularly in biology, medicine and social sciences no mathematical model is known or, even if a model is known, it still has many unknown and not measurable ingredients. But, in most of these problems we still can collect large amounts of data from experiments or observations. Then, instead of a differential equation we have a cloud of vectors that is a cloud of points with a vector attached to each point. One can try to find a differential equation which fits the cloud of vectors but such an equation is artificial and becomes discrete again when studied by means of numerical methods. Therefore, it is natural to ask whether a combinatorial counterpart of a classical dynamical system may be constructed on the polyhedron approximating the cloud and whether such a combinatorial dynamical system may be investigated by means of analogous topological tools as in the case of classical topological dynamics. A partial answer to this question is in the papers from the end of the 20th century and the beginning of the 21st century written by American mathematician Robin Forman. He constructed a combinatorial analogue of Morse theory, a fundamental theory in differential topology constructed in the first half of the 20th century by another American mathematician Marston Morse. Conley index theory may be considered an extension of Morse theory towards the needs of the asymptotic study of dynamical systems.

Taking combinatorial Morse theory by Forman as starting point we will develop a general theory of combinatorial dynamical systems together with an appropriate combinatorial analogue of Conley index theory and utilizing Alexandrov topology on finite topological spaces. We will design algorithms which will let us construct combinatorial dynamical systems for dynamical data. We will implement these algorithms together with algorithms performing global analysis of combinatorial dynamical systems by means of topological invariants. We will test the algorithms on the simulations of selected practical problems. Within the project we will closely collaborate with top European and American scientists working on related problems.

The outcome of the project will be an entirely novel technique for building mathematical models from dynamical data. This will enable new methods for the analysis, classification and prediction of sampled dynamical systems. The new theory will be accompanied by an open license software library for constructing the models from data and analyzing it. The results will be published in top international journals and presented in international conferences.