Fundamental aspects of the quantum set of correlations

Quantum information theory is an active research field at the intersection of physics, mathematics and computer science. The principal goal is to understand whether quantum systems, e.g. single atoms, photons or electrons, can be used to gain advantage in information-processing tasks like computation, transmission or cryptography.

One particularly counterintutive aspect of quantum mechanics concerns the correlations that can arise between space-like separated systems. This is precisely the Einstein-Podolsky-Rosen paradox raised in 1935, which led the authors to the conclusion that the quantum formalism should not be considered complete. This was later formalised by John Bell, who proved that a certain kind of quantum correlations, which we now refer to as *nonlocal correlations*, cannot be reproduced by any classical (local-realistic) theory. To make the distinction between quantum mechanics and classical theories mathematically precise, it is convenient to define the concept of a *correlation set*. The correlation set for a given theory (or a class of theories) is defined as the union of all correlations achievable within that theory. The most important sets of correlations correspond to classical theories, quantum mechanics and no-signalling theories. Classical theories are those which follow our everyday intuition that the act of performing a measurements is a passive action that simply reveals a pre-existing value. No-signalling theories are those that respect the principle that performing a measurement in one region should not have an immediate effect on a region that is space-like separated. It turns out that the both these sets are rather simple: they are polytopes, i.e. convex sets with a finite number of extremal points. This means that both of them admit a closed-form, convenient to work with description and their properties can be evaluated algorithmically. The quantum set, which sits in between these two sets, is not a polytope and no efficient description (beyond the definition) is known. Our main goal is to investigate some fundamental features of the quantum set.

The research tasks of this project include: parametrising all the extremal points in the simplest non-trivial Bell scenario, finding the simplest scenario in which infinite-dimensional quantum systems outperform finite-dimensional quantum systems, studying computable relaxations of the quantum set and understanding the relative size of the three sets mentioned above.

In this project through a mixture of rigorous analytical tools and numerical studies we will advance our knowledge and understanding of the fundamental aspects of the quantum set of correlations. Although this project is predominantly of foundational nature, we hope that our findings will also contribute towards the development of practical quantum technologies.