The proposed research aims not only at solving very hard problems in mathematics; it will also advance the field of computer–aided mathematics, also known as *validated numerics*. Here, we are not simply using the computer to produce numerical approximations (of unknown quality) of a dynamical system; we are using the computer to *verify* mathematical properties of the system.

When it comes to continuous problems like the ones we are addressing, this is a true shift in paradigm: we are now entering an era when very advanced numerical computations can come equipped with a mathematical certification. All necessary error bounds are done on–the–fly along with the computation itself. This additional information can be used to adaptively modify the computations as they are performed: the level of discretization of the problem can be fully controlled, and locally adapted to the problem. This fundamental change results in efficient and extremely fault-tolerant numerical methods, ideal for the systematic study of nonlinear systems.

It is our belief that validated numerics will play an instrumental role as computers become increasingly dominant in every–day life, as well as in scientific research. The more we rely on numerical computations, the more we must be able to trust them. This is important in many applications such as control of industrial robots, and in the basic research stages of drug development. And from the mathematician's point of view, the ability to use numerical computations as an integral part of a proof opens up a huge range of problems that have previously been out of reach.

The theories and techniques of validated numerics define a new research area, along with a new breed of scientists equipped with the combined strengths of pure mathematics and scientific computing.

Our aim is to apply this approach to the problems in celestial mechanics. We will address the problem of the number of relative equilibria in the n-body problem, which is one of Steven Smale's problems for the 21st century century. We will also consider the problem of oscillatory motions, that is motions of asteroids coming within a planetary system and then bouncing back towards infinity in a chaotic manner. We will also apply the approach to investigate the dynamical properties of orbits of asteroids which terminate by colliding with a star, a planet or a moon.