Set theoretic methods in dynamics and number theory (Description to the general public)

Our primary goal is to study objects appearing naturally in dynamics and number theory from the perspective of descriptive set theory. Sets are classified according in the *Borel hierarchy* which gives a natural measure of their complexity. In a more precise sense, the complexity of a set is determined by the number of alternating quantifiers ("there exists" and "for all") over certain infinite sets. Knowing the Borel complexity of certain sets can offer insights into various phenomenon or help to solve certain problems.

One set that has been of recent interest from a descriptive set theoretic perspective is the set of numbers normal in base b. Informally, a number is normal in base b if its b-ary expansion is "random" in the sense that each digit $0, 1, \ldots, b-1$ occurs with frequency $1/b$, each pair of digits $00, 01, \ldots, (b-1)(b-1)$ occurs with frequency $1/b^2$, and so on. H. Ki and T. Linton determined the exact complexity of the set of numbers normal in base b and V. Becher and T. Slaman have recently solved a couple of related open problems of A. Kechris.

We are also motivated by problems in dynamics. The problem of H. Ki and T. Linton is really just a special case of a more general problem to determine the complexity of the set of generic points for a given dynamical system. A recent result of D. Airey, S. Jackson, D. Kwietniak, and B. Mance showed that the result of H. Ki and T. Linton generalizes to a class of dynamical systems with a property called *specification*. It is already evident that many systems without this property have a simpler set of generic points. The project will further investigate complexity of sets naturally arising when studying interval exchange transformation, Poissonian pair correlated sequences, and in other contexts of interest when studying dynamical systems and number theory.

The main technique for these investigations is called *Wadge reduction*. Wadge reduction is more of a meta-method allowing one to transfer the structural propeties from the domain space (where they are well understood) to the range space where there is often no desription at the structural set-theoretic level. Therefore, to construct a reduction you often need new tools or theorems that illuminate necessary structure and allow one to define a map with the desired properties.

Also, it seems that knowledge about the set of normal numbers for a very general class of expansions called a Cantor series expansion may provide geometric information about certain classes of curves called algebraic varieties. A recent result of D. Airey and B. Mance suggests that this connection may be possible and we wish to investigate further. Additionally, the project will investigate the transcendentality of number normal for Cantor series expansions motivated the recent work of Veekesh Kumar.