Popular science description

In view of recent experimental result, classical diffusion models fail to describe many biological and physical complex systems. The current challenge is to find the proper mathematical model for the sub- and superdiffusion observed in many complex systems, and to apply this knowledge so that we can manipulate and control the dynamics of the process. Continuous time random walks (CTRWs) have a special position among the models of anomalous diffusion. From the mathematical point of view CTRWs have a rich structure, which allows to link notions from different branches of theory of stochastic processes and fractional integro-differential equations. Physically, they combine the theory of random processes with statistical physics and allow to describe the motion of single particles. This rich structure also strengthens the practical usage of CTRW and related fractional equations in different areas, such as biology, chemistry and financial engineering.

In the proposed research project we will consider three fundamental aspects of anomalous diffusion processes – probabilistic, numerical and statistical. Our research will develop rigorous mathematical theory in connection with advanced numerical and statistical methods, in order to identify and verify proper origins of anomalous dynamics experimentally observed in complex systems.

We plan to build the mathematical theory of certain classes of CTRWs. The choice of these models is strongly motivated by the results of single particle tracking experiments. The emphasis will be put on the so-called CTRWs quenched in space and aging Lévy walks, which will allow to model the inhomogeneities in living cells. We will derive certain limit theorems for aging Lévy walks and CTRWs quenched in space in different configurations. The detailed description of these scaling limits will let us model the behavior of corresponding complex systems for long times. We will also build the numerical and statistical methods in order to analyze anomalous diffusion processes described by the so-called fractional Fokker-Planck equation, which is the most important equation of subdiffusive dynamics. Our results can be used to identify the origins and mechanisms of anomaly in various complex systems. Finally, we will perform extended numerical simulations in order to to verify the performance of our methodology.

This interdisciplinary research project should contribute significantly to the better understanding of the anomalous diffusions and fractional dynamics. The theoretical results related to limit theorems for CTRWs can help to extend the mathematical description of anomalous dynamics and to find its proper physical interpretation, whereas the applicational part will give numerical and statistical methods appropriate for the analysis, verification and estimation of the parameters of anomalous complex systems. These up to date issues, which are at the moment the subject of studies of various leading scientific centers (Cambridge, Heidelberg, MIT, Princeton, Tel Aviv, München), will have considerable impact on mathematical, physical and biological sciences.