Summary

Measure data problems

In recent years, a great role is played by modelling of modern materials with complex structure such as composite materials having grains of various densities, thermo-visco-elastic ones changing viscosity depending on temperature and their structure, non-Newtonian fluids changing rapidly density while exposed to external force (used for example in production of bulletproof vests), electrorheological fluids changing properties while exposed to electric field (used in intelligent braking and shock absorbing systems). Classical theories of elasticity and fluid mechanics are expressed by the use of nonlinear partial differential equations under assumption that the involved nonlinearities are close to polynomials, but it is not sufficient to describe physical phenomena of inhomogeneous materials. In order to model such substances and phenomena it is necessary to admit operators with nonstandard growth, and this leads to problems stated in more unconventional functional framework like anisotropic Orlicz spaces or inhomogeneous and anisotropic Musielak-Orlicz spaces. Analysis in this setting is an object of intensive study in Finland, Korea, Japan, Germany and Italy.

The aim of the project is investigation on properties of the involved functional spaces, which are needed in study on mentioned models. In particular, in order to stress the meaning of the results we plan to apply them in analysis of elliptic and parabolic problems with nonstandard growth, posed in the mentioned generalized spaces. Besides existence and regularity of various notions of very weak solutions to measure-data problems, we shall be interested in potential estimates for them and their regularity. The methods we plan to develop will enable for analysis of the mentioned models in far broader context than it was so far possible – the point is not only to capture by unified theory many known cases, but also deeper understanding of the nature of problems. In particular, investigations on equations in spaces changing along time are a big challenge, but they are also pioneering and it is a very promising direction. There are many open questions in the regularity or potential theory that received only a partial answer in the variable exponent spaces. We want to embrace them, deepen, and generalize to the more general setting.