

This is a fundamental research project in pure mathematics, specifically in differential geometry. For more than a century this area was dominated by Riemannian geometry, which is a curved version of Euclidean geometry familiar from school. There is an abundance of other geometries based on different notions than distance, namely the geometry of conformal structures (angles), of geodesics (shortest lines), of constraint velocities (nonholonomic mechanics), of differential equations (evolutionary dynamics), etc. They have a collective name of Cartan geometries. Some of these geometries find applications in alternative theories of gravity and other physical theories, but the majority existed as mathematical abstractions. The goal of the project is to investigate a uniform broad class of Cartan geometries with the new tools developed in mathematics in recent decades.

The project is called SCREAM because its central themes are Symmetry, Curvature Reduction, and EquivAlence Methods. Symmetry is fundamental in all natural sciences and is tantamount to niceness of the geometric structure. Curvature is a mathematical counterpart of gravity and it is a mechanism of Reduction of Symmetry. EquivAlence is a way to make Equal or Align seemingly different geometric structures, and we will exploit various Methods for this purpose. An example one can keep in mind is that of distant stars, observed by independent astronomers, and identified using mathematical methods.

We seek to implement and refine the techniques of Cartan geometries in order to answer questions of fundamental importance for a variety of geometric structures beyond the classical setting. This concerns computing the symmetry size, establishing existence of solutions to physically motivated equations, recognizing geometries via their invariants, and investigating geometric robots whose configuration spaces support fascinating geometric structures. Applications include optimal control, general relativity, cosmology, integration of equations of mathematical physics and much more.

These general geometries are inspired by the work of Elie Cartan, a French mathematician with a strong influence on Polish theoretical physics. His achievements were in turn based on the works of the Norwegian mathematician Sophus Lie, unifying algebra and geometry. The project will conduct an international cross-disciplinary collaboration. We aim for a group of Polish and Norwegian experts, assisted by young researchers, to make fundamental contributions in a novel investigation of a wide class of geometries with modern techniques, equipments and methods.