# Interaction in quantum field theory

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Imagine an electron colliding with a positron in a particle accelerator. What is the probability that a Higgs particle will be produced in this collision? The theoretical framework physicists use to answer such questions is called quantum field theory (QFT). The usual computational method within this framework gives an answer in the form of an infinite sum of successive corrections. By considering only first few terms of this sum, one often obtains a very good agreement with experiments. However, if we could sum up all the terms, the result would most likely be infinity. Can we claim agreement of theory and experiment in this situation?

In the light of the above dilemma, the problem of logical consistency of interacting QFT in four-dimensional spacetime remains open to date. This fact is at the basis of the *Yang-Mills Existence and Mass-Gap Millennium Problem* of the Clay Mathematics Institute which concerns a rigorous mathematical construction of the Yang-Mills theories. The physical context of this problem is clear: these QFTs are the main building blocks of the Standard Model of particle physics. But on the mathematical side it is legitimate to ask what actually should be constructed.

To answer such questions, already in the 1950s the first axiomatic systems for QFT have been formulated. They give a precise mathematical meaning to very general principles such as stability of matter, independence of a choice of an inertial reference frame or mutual independence of measurements performed in spacelike separated regions. A good news was that such general assumptions suffice to consistently define probabilities of collision processes. However, natural assumptions ensuring the presence of non-trivial physical processes, involving interaction between particles or particle production, are not known. It is also not known, after five decades of research, if there is any interacting QFT in four-dimensional spacetime, fitting into the axiomatic settings mentioned above.

While we are not going to construct the Yang-Mills theories, the goal of this project is to clarify the problem of interaction in axiomatic QFT and to explore some new approaches to construction of interacting models.

Objective (A) of this project is to exploit an unexpected relation between interaction in QFT and a non-ergodic behaviour of a certain auxiliary system. Ergodicity means that averages over time of certain physical quantities are equal to averages over a time-independent statistical ensemble. On the side of QFT, this time-independence means that configurations of particles do not change in physical processes, thus there is no interaction. By proving suitable *ergodic theorems*, we will restrict the realm of interacting theories.

Objective (B) is to construct a class of QFT in two-dimensional spacetime, which arise by restricting an $N$-element family of free fields to the unit sphere. These so called $O(N)$ non-linear sigma models have many properties in common with the Yang-Mills theories or the Heisenberg ferromagnet. A construction of the non-linear sigma models is an important open problem in mathematical physics. Our approach relies on recent seminal works of M. Hairer (Fields Medal 2014) on stochastic differential equations. The method of stochastic quantization relates solutions of such equations to probabilities of physical processes in QFT. Apart from a construction of flat (Minkowski) spacetime we also plan to construct the non-linear sigma models on curved (de Sitter) spacetime.

Objective (C) is to look at other QFT which arise by imposing constraints on free theories and search for interacting models in four-dimensional spacetime within this unexplored class. A particularly promising example is given by a positivity constraint on the field, as it seems tractable by explicit computations. Namely, probabilities of physical processes can be reduced to integrals of polynomials over certain subsets of a multi-dimensional sphere. In parallel, we will study this model using the method of stochastic quantization mentioned above.