Chaos, fractals, and not only conformal dynamics

The dynamical systems are systems that change their state in time, each and every mathematical model in physics and other sciences is a dynamical system. For a given initial state of the system we try to predict its future, especially long term future.

Naturally, the futures of different initial states are different. However, very often we can distinguish between the 'typical' behavior of the system and 'atypical' behavior, which can also happen for some initial states but is much more rare. Those rare events can have, however, very far-reaching consequences if they *do* happen. The problem is that those rare initial states are not necessarily distinguishable from typical ones – looking at the initial state of a dynamical system we might not be able to say its exact future. We can, however describe it in probabilistic terms: we can say, for some given a priori probabilistic distribution on the phase space, how probable is that certain behavior will happen.

The goal of our project is to develop this qualitative theory of rare events in some classes of dynamical systems. We want to investigate the behaviors of ergodic type, we will also investigate geometric behaviors – for example, asking about initial states which trajectory will never visit some area of the phase space. Using weather as an example, the former question would be asking about the average temperature over the next 100 years while the latter would be about chances of not having even once in this century any snow in Warsaw on New Year Day.

Obviously, we are working on much simpler models than the weather. We will investigate different classes of one-dimensional systems, both real (iterated function systems with finitely and infinitely many branches, irrational rotations) and complex, as well as some special classes of higher-dimensional dynamics (affine maps, some diffeomorphisms in $\mathbb{R}^3$) and matrix cocycles. We will estimate the 'size' of the atypical trajectories sets using Hausdorff dimension and topological entropy.