Algebraic geometry is a branch of mathematics which uses geometric methods in algebraic problems and algebraic methods in geometry. The most important object of study in algebraic geometry is an *algebraic variety*: a geometric object described by polynomial equations. One of the most spectacular applications of algebraic geometry is the proof of the *Fermat Last Theorem* formulated in 1637 which states that for an integer $n > 2$ the equation

$$a^n + b^n = c^n$$

does not have a non-zero solutions in integers a, b, c. G. Frey observed that the congruence

$$y^2 \equiv x(x - a^n)(x + b^n) \pmod p$$

would have an unexpected number of solutions, for a, b satisfying Fermat's equation. A set of pairs of complex numbers satisfying a similar equation:

$$y^2 = x(x - a)(x - b)$$

is called an *elliptic curve*, completed with a distinguished "point at infinity" it is a torus. The existence of non-zero solutions of the Fermat equation would imply that the elliptic curve associated to that solution does not satisfy the Taniyama-Shimura-Weil conjecture. In 1993, A. Wiles proved the Taniyama-Shimura-Weil's conjecture and consequently Fermat Last Theorem.

The main object of the present project are higher dimensional counterpart of elliptic curves called *Calabi-Yau manifolds*: projective manifolds of dimension $n > 1$ having a complex volume element, but not admitting a non-zero holomorphic differential k-forms, for $0 < k < n$.

The interest in the Calabi-Yau manifolds is strongly motivated by applications in theoretical physics, especially in the so-called *string theory.* The string theoretical model of the universe is a 10-dimensional real variety, which can be "compactified" to the product $M^4 \times X$ of the "usual" Minkowski's spacetime M^4 and a Calabi-Yau manifold X.

Many string theoretical observations lead to deep mathematical consequences. The most famous mathematical problem inspired by the string theory is the *Mirror Symmetry Conjecture*, according to which Calabi-Yau threefolds should appear in pairs satisfying the following condition

$$h^{1,1}(X) = h^{1,2}(Y) \quad \text{and} \quad h^{1,1}(Y) = h^{1,2}(X),$$

where $h^{i,j}$ are one of the most important numerical invariants of algebraic manifolds, called *Hodge numbers.* Despite of intensive studies, many basic questions concerning Calabi-Yau manifold including classification are still open.

There are many methods of constructing Calabi-Yau manifolds. The toric construction due to Batyrev and Borysov provides the largest number of examples. In the presented project, we intend to study two constructions that can be considered as a direct generalizations of an equation of an elliptic curve: double octics and fiber products of elliptic surfaces. In particular, we plan to study "modifications" of these constructions by considering quotients of these varieties by a finite group acting on them. One of the most serious difficulty occurring in that quotient varieties is the existence of singularities corresponding to points with non-trivial stabilizers. According to Hironaka's desingularisation theorem, complex manifolds admit a resolution of singularities, i.e. a modification without singular points.

Resolution of singularities which produces Calabi-Yau manifold need to preserve a canonical divisor, we call such a resolution *crepant.* The existence of crepant resolution is well studied only in the case of the variety of the dimension ≤ 3.

For computation of the Hodge numbers of constructed manifolds we shall use the *Chen-Ruan orbifold cohomology theory.* As one of the results of this project we expect new examples of Calabi-Yau manifolds that are modular, which means that they satisfy analogous Taniyama-Shimura-Weil's conjecture in the dimension ≥ 3.