Description for the general public

The main task of a structural engineer is designing a structure (within a given design domain) which the self-weight and the given traction load applied on the given surface transmits to the supports or to the place where the supports are planned. The subject of the optimization (or the minimization operation) is the volume of the material used or the overall compliance. The optimum design is chosen from the structures of a given material cost. The problems thus formulated belong to the class of problems of *structural topology optimization* (STO) which gradually replaces the parametric optimization.

Till now the three methods (or rather three groups of methods) have been developed which reduce the STO problems to correctly set mathematical problems. In the first method (1) we assume that the unknown structure is a framework; the aim of the optimization is minimization of the compliance, while the cost is proportional to the total volume. This problem is mathematically equivalent to Michell's problem of minimization of the volume with the conditions bounding the stress level by the equal values of permissible stresses in tension and compression. The method (2) concerns structures composed of two isotropic materials; the aim of the optimization is compliance minimization, while the cost is equal to the volume of one of the constituents. The method (3) deals with designing the elastic moduli; the objective function is the compliance and the unit cost is assumed as the sum of the principal elastic moduli. To make the method (3) belong to the class of STO, one introduces only one, the weakest, condition on the energy density: it cannot assume negative values.

All the mentioned above STO methods have the built-in mathematical (not numerical !) algorithm of *cutting out the material domain from the design domain*. The method (1) determines the position of nodes and bars. The method (2) cuts out the optimal structure and determines juxtaposition of the constituents. The method (3) also cuts out the optimal structure and determines the distribution of the material characteristics.

The first two research objectives refine the available method (3) in its linear elasticity setting. The aim of the refinement is eliminating singularities of the optimum design expressed by abrupt growth of the values of optimal moduli at the places of stress concentrations. An improvement can be achieved by introducing yield conditions which bound some scalar characteristics of the stress state; on the other hand, the method (3) does not lose the attribute of the STO method: possibility of *cutting out the material domain from the design domain*. The subsequent planned refinements take into account sensitivity of the material response to the sign of stress or they describe specific features of the state of stress in the reinforced concrete, in masonry structures and in structures made from the fiber-reinforced composites with fibers that cannot transmit compression stresses. It occurs that this refinement preserves the mentioned feature of cutting out the material domain from the design domain. This feature is built-in in the auxiliary problem of the method (3) in its stress-based version. To make the numerical methods as effective as possible we plan to complement the stress-based auxiliary problems with the displacement-based formulations to form the so-called dual pairs of problems of zero duality gap. Thus, the optimum designs will be constructed parallel in two manners to be sure that the final result is correct. These auxiliary problems possess the mathematical structure of the optimal transportation problem; indeed, the operation of minimization over the elastic moduli lead to new problems, expressed by the equations of mathematical physics of the form by far different from the original problem of structural or continuum mechanics.

The planned research tasks deliver the answers to the questions of how to design a structure from the available amount of the material (obeying the elasto-plasticity laws or sensitive to the signs of stresses) to make it possible to optimally transmit the given load to the given support. The original versions of the method (3) were based on the assumption of small elastic deformation. Other, available, not described here approaches are numerical simulations; they are expressed in terms of the finite element discretization. However, the operations of optimization and discretization do not commute. One of the aim of the project is a correct extension of the method (3) to preserve its continuum approach and not preceded by a discretization process.

The third objective concerns designing the roofs which are uniformly compressed, the roofs covering a given planar domain. In the past the physical models of such roofs had been made from ropes with weights; the visitors of the museum in Sagrada Familia church in Barcelona are impressed by such hanging structures designed by Antonio Gaudi. Yet these models are not correct as attempting to form a spatial structure from simple funiculars. A correct method of creating optimal hanging nets over given planar domains is delivered by the Michell method with using the 3D ground structure method and the recently discovered variational methods of construction of Prager-Rozvany archgrids designed for transmissible loads. This is the third objective of the present project.