Popular science summary

The project concerns important areas of Fourier analysis and operator theory with applications to partial differential equations. The main goal of the research will be to verify difficult hypotheses and answer questions posed in the project. Proving these results will make valuable contributions to Fourier transform theory, the theory of multidimensional Fourier series, Boolean function analysis, interpolation of operators, ideals of Lipschitz operators on metric spaces, operator Calderón–Zygmund theory, the local theory of Hardy spaces of Dirichlet series and develop a theory of s-numbers for multilinear operators. The research will concern the following topics, with a broad research spectrum:

- The multipliers of bounded Boolean functions on the infinite-dimensional cube
- Arithmetic diameter, Sidon sets and multipliers of the Hardy space of bounded Dirichlet series
- The Fourier transform between function spaces with applications to the Schrödinger equation
- Almost everywhere convergence of multiple Fourier series
- Calderón-Zygmund singular operators with applications to PDEs
- Interpolation of bilinear operators by the method of means
- Stability of isomorphisms in interpolation spaces generated by the method of means
- s-numbers and ideals of multilinear operators
- Lipschitz (q, p)-summing operators on metric spaces

We will describe only some of the basic objectives related to the above research tasks. We plan to prove new results in the areas indicated. Some of our results obtained, in the last period, on Boolean functions defined on the finite-dimensional Boolean cube are, in some sense, the source of new, original, non-trivial hypotheses. In connection with this, we plan to develop some ideas based on Fourier analysis on Abelian groups and the Grothendieck–Pietsch factorization theorem to analyze functions on the infinite-dimensional cube. It can be expected that the solution to these hypotheses will make an important contribution to the theory of Boolean functions. A particularly important goal of the project will be to prove the hypothesis of convergence in the area of almost everywhere multiple Fourier series over polytopes of functions from the Arias de Reyna space on a multidimensional torus. The solution to this hypothesis would certainly be a spectacular achievement and would improve the best known results so far, in particular the result by C. Fefferman. In the scope of the subject connected to Hardy spaces, a key task will be to apply the methods of harmonic analysis and the local theory of Banach space to study the local structure of the Hardy space of bounded Dirichlet series and the space of multipliers for this space. In the area of interpolation theory the main goal will be to prove theorems about the stability of isomorphisms on interpolation spaces generated by the method of means, and about interpolation theorems for two-line operators between these spaces. An important field of research will be the study of Fourier transforms and Fourier multipliers between Banach function spaces and their application to the solution of the differential equation of Schrödinger. We also will study Calderón–Zygmund singular operators between function spaces and plan to show applications for partial differential equations. An important goal of the project is to build a fundamental foundation for theory of s-numbers for multilinear operators and initiation of the study of Lipschitz (q, p)-summing operators for $1 \leqslant p < q < \infty$. Here one of the key aims will be to prove a non-linear version of a deep factorization theorem of Pisier for this type of operators from $C(K)$-spaces to complete metric spaces. The main reason for undertaking research topics in the areas of Fourier analysis and the theory of operators is its originality. The topic related to the Boolean functions are motivated by important applications of these functions, including in quantum information theory, signal theory and graph theory. Within the scope of the research topic related to Fourier analysis the problem of convergence almost everywhere is one of the fundamental problems in the area. Proving new results associated with this problem will certainly make a valuable contribution to Fourier series theory. The theory operators is an important part of modern functional analysis, which has numerous applications in many areas of mathematics. This fact is a solid motivation to undertake research in this area that can be successfully concluded and used in spectral analysis and the theory of partial differential equations.