

This project aims to provide a revision of the current views on the foundations of the calculus in terms of relationship between intuition of space and motion and the mathematical concepts of limit, continuity and function. Our project is interdisciplinary: both philosophical and mathematical. We challenge Rafael Núñez's standpoint on the role of intuition in the development of the calculus.

While the standard history of the calculus derives the arithmetization program from Cauchy's *Cours d'Analyse* (1821), our arguments are based on the new interpretation of Bernard Bolzano's mathematical writings, primarily Bolzano's *Rein analytischer Beweis* (1817) and Bolzano's *Paradoxien des Unendlichen* (1851). It is because, firstly, Bolzano exemplified the duality between real and nonstandard analysis. In (Bolzano, 1817) he developed foundations for real analysis, in (Bolzano, 1851) he applied techniques that currently can be easily interpreted in nonstandard analysis. Secondly, Bolzano also marked the split of Greek concept of continuity. Thirdly, Bolzano's definition of continuous function has motivated our study of a gap between logical structure of natural and symbolic language.

Núñez challenges the claim that "arithmetization of calculus marked a shift away from spatial-dynamic intuitions, grounding concepts in static, rigorous definitions". His arguments can be summarized as follows: (1) limit, continuity and function are the essential concepts of the calculus; (2) there is a "natural", "dynamic conception of continuity" which characterized mathematical thinking up until the end of the first decades of the 19th century. Still, the "natural continuity" is also "consistent with the idea of continuity as understood outside of mathematics, drawing on basic, shared intuitions of space and motion"; (3) evocative language used by both professional mathematicians and students, identified in teaching and learning of the calculus, is to compensate for a gap between "static, rigorous definitions" and "intuition of natural continuity".

We aim to revise these three basic claims to showing that: (ad 1) there are mathematical theories undermining Núñez's model; (ad 2) Núñez's "natural continuity" is in fact the ancient Greek concept of continuity; (ad 3) evocative language revealed by Núñez results from the gap between logical structure of the definition of continuous function as given in a natural language and its symbolic counterpart.

This project is important to philosophy and foundations of mathematics. We provide the assumption of geometrical period of the calculus. We demonstrate that Núñez's "natural continuity" is just the ancient Greek notion of continuity. We show that two different notions of continuity which characterize modern mathematics originated from (Bolzano, 1817).