We study a number of related objects like affine stochastic recursion (**ASR**), branching processes in random environment (**BPRE**) and random walks in random environment (**RWRE**). They have numerous applications to financial mathematics, biology or physics etc. Although all of them have been considered for several dozen years and many aspects are quite well understood, there are still challenges ahead.

Affine stochastic recursion is one of the best known examples of a Markov chain. It appears in financial mathematics e.g. ARCH, GARCH and BEKK models. It is used to construct classical fractals as Sierpiński gasket or Barnsley fern. ASR is often presented as a particular example of an iterated function system e.g. in the well known book by Barnsley. A little bit more general stochastic difference equations (**SDE**) are applied to population dynamics. Affine stochastic recursion appears also in purely theoretical mathematics for instance in study of Poisson boundaries for harmonic functions.

In the last few years we put a lot of effort to understand global properties of stationary solutions to affine stochastic recursions, e.g. decay of their tails. Now we intend to study local regularity, fractal properties. Currently a number of mathematicians Breuillard, Hochman, Shmerkin, Varju study "so called" Bernoulli convolution, a very particular example of an affine stochastic recursion. Due to the recent development in the subject quite a detailed description of its fractal properties has been obtained. **Within the project we intend to go beyond Bernoulli convolution and to study absolute continuity of stationary measures in the general case taking into account many other examples.**

SDE may appear when fragmentation is considered. It is a phenomenon of breaking up particles into a range of smaller sized particles, characteristic of many natural processes ranging from e.g. polymer degradation to breakage of aggregates in plankton dynamics. Being determined by a number of conditions fragmentation is not that easy to analyze. It may be approached through transport equations and then finding self-similar solutions is highly desirable. **We would like to use SDE as a tool to obtain such solutions**.

Branching processes provide a model of population growth. The simplest and the most known is "so called" Galton-Watson process, when the populations grows in an environment constant in time. We are going to study a model that is more realistic i.e. the environment changes in time. It is called a branching process in random environment. As an example one can think of a population of plants with one year life-cycle. Each year the environment (climate) varies and influences the reproduction mechanism.

Basic questions concerning branching processes are: probability of extinction and probability of **deviation of the population size from the mean**. The former is well understood. **We are going to study the latter and we want to obtain better asymptotics that currently known. In particular, we are interested in the multitype case i.e. when the individuals in the population have different properties and so we have a "vector" of children.**

Standard random walks provide a discrete model for transport processes like propagation of heat or diffusion. However, for many practical reasons an assumption that the environment is homogeneous is not appropriate. One should consider models where the medium in which the process lives is irregular and reflects random factors like defects and fluctuations. This brings the notion of a random walk in a random environment. The latter means that the probability of going in certain direction depends randomly on the spot.

We are going to study asymptotic properties of **RWRE**. As for **BPRE** fluctuations from the average behaviour are of issue. **A powerful method of studying RWRE is to compare them with BPRE with immigrants and finally with the affine stochastic recursion. This method allows to reduce problems concerning RWRE to ASR, use the tools we have already mastered and study fluctuations of the walk from the typical behavior.**