RESEARCH PROJECT

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Our project is connected with geometry of polynomial and semi-algebraic mappings. We consider four issues:

1) **Extension of bi-Lipschitz mappings** Let X, Y be a metric spaces (mainly subspaces of $\mathbb{R}^n$). We say that a bijective mapping $f : X \to Y$ is bi-Lipschitz if there is a constant $K > 0$, such that

$$\frac{1}{K}|x - y| < |f(x) - f(y)| < K|x - y|,$$

for every $x, y \in X$. Study of such mappings (the Lipschitz geometry) nowdays is an important part of singularity theory and algebraic geometry. Using the method of Lipschitz geomery we can classify algebraic complecx and real sets. We are going to prove that a semi-algebraic set X of dimension k has only one bi-Lipschitz embedding in $\mathbb{R}^n$, if $n > 2k + 1$ (up to a bi-Lipschitz homeomorphism of $\mathbb{R}^n$).

2) **Stability of polynomial mappings** Classical method of singularity theory can not be applied for algebraic families of polynomial mappings. Indeed the Whitney topology is discrete on any of such family. In our previous project we have started to develop singularity theory for families of polynomial mappings. In particular we have proved a version of the classical Whitney theorem for the family $\Omega_{\mathbb{C}^2}(d_1, d_2)$ of polynomial mappings $f : \mathbb{C}^2 \to \mathbb{C}^2$ of fixed degree. In the project we will define the notion of stability of a polynomial mapping and we will investigate its properties.

3) **Generic symmetry defect set of an affine variety**

Over the last two decades numerous methods have been developed to study affine geometry of surfaces and curves. The symmetry sets and the center symmetry sets were investigated extensively for real smooth ovals. In our project we will build an algebraic counterpart of this theory and we will define a notion of a generic defect symmetry set for algebraic variety $X^n \subset \mathbb{C}^{2n}$. We will show that it has nice singularities and we will describe its geometry for generic algebraic curves of fixed degree.

4) **Topological classification of quadratic polynomial mappings** $F : \mathbb{C}^3 \to \mathbb{C}^2, F : \mathbb{C}^3 \to \mathbb{C}^3$.

In our previous project we have classified topologically quadratic polynomial mappings $F : \mathbb{C}^2 \to \mathbb{C}^n,\ n = 1, 2, 3, ...$ Here we would like to classify topologically quadratic polynomial mappings $F : \mathbb{C}^3 \to \mathbb{C}^2, F : \mathbb{C}^3 \to \mathbb{C}^3$. This is very difficult problem, but we believe that at least some partial results are possible.