Geometric quantities related to nonlocal equations

Description for the general public

In the framework of the research project we intend to study geometric quantities that are defined via solutions to a certain class of nonlocal partial differential equations. A prominent example is the perimeter of a given measurable subset of the Euclidean space $\mathbb{R}^d$. In the classical (local) setting perimeter of a set with smooth boundary is defined to be the Hausdorff measure of its boundary. This definition can be generalized for a larger family of sets (without smooth boundary) by employing a concept of bounded variation. In the nonlocal setting we imitate this approach and define perimeter of a set as a certain functional which measures the boundary behaviour of the set in terms of a given function or measure (including Lévy measures). Within the project we shall study not only nonlocal versions of perimeter for a rich class of sets but also an intimately connected notion of heat content and a related geometric object called mean curvature. We are focused on the connections to their classical counterparts (approximation results) and on their geometric and analytical properties, like isoperimetric inequalities and asymptotics.

Nonlocal versions of the classical geometric objects as mentioned above play an important role in many fields of natural sciences. For instance, nonlocal perimeters may be successfully applied to recover the perimeter of the original image in processing of the BMP-type images of high resolutions. Nonlocal curvature appears to be a useful notion in the fluid dynamics and phase transition models, including problems of minimal surfaces and motion by the mean curvature. It has also significant importance in theory of interfaces in gases (or solids) and in the theory of superconductors and superfluids.

Another aspect of our project concerns the free-boundary problems of Bernoulli type for star-shaped sets which find numerous applications in such areas as plasma physics, semi-conductor theory and flame propagation. Again we are mainly interested in the nonlocal framework, namely we plan to work with the fractional Laplacian which comes up naturally if long range interactions are taken into account. The main objectivities of our research are to investigate the existence and uniqueness of solutions, which heavily depend on the geometric properties of the domain on which the Bernoulli problem is stated.

Summing up, our project is an interdisciplinary research which crosses mathematics and physics. Our methods and approaches constitute a mixture of analysis, probability and geometry and thus the discussed topics should attract a lot of attention of researchers from various fields of science.