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Structure of random permutations and the interchange process

project description for the general public

Michał Kotowski

Let us imagine we have a deck of N cards and we want to shuffle by the following procedure: at each step, we choose two cards uniformly at random and swap their positions. How many steps do we need to perform until the cards are well mixed? What is the typical structure of the resulting permutation of cards?

Card-shuffling models like the one just described (the *interchange process* one the complete graph) are a classical object of study in the theory of Markov chains and their applications. What is somewhat surprising is that similar models are closely related with fundamental questions of statistical physics, in particular the study of phase transitions in quantum spin systems.

A well-studied model of ferromagnetism is the *quantum Heisenberg ferromagnet*, which is conjectured to undergo a phase transition at low temperatures – below certain critical temperature, the system exhibits nonvabishing spontaneous magnetization. It turns out that analogous phase transitions happen in the models of *spatial random permutations* like the interchange process – when sufficiently many transpositions are performed, suddenly long (macroscopic) cycles appear in the cycle structure of the permutation.

This project is situated at the boundary between probability theory and mathematical physics. We aim to study the cycle structure of various models of random permutations, in particular the interchange process on graphs with complex geometry. Apart from the mathematical beauty of the subject, our motivation is to shed light on important questions in statistical physics. In particular, we would like to make progress on the famous conjecture due to Bálint Tóth, stating that the interchange process on the d-dimensional lattice $\mathbb{Z}^d$ in dimensions $d \geq 3$ should exhibit a phase transition in the cycle structure.

We hope that this project, bridging classical topics from the theory of Markov chains and phase transitions in physics, will result in a deeper understanding of spatial random permutations, currently a very actively studied subject, and their *universality* – the phenomenon of different models of permutations exhibiting the same asymptotic behavior.