

# Exact, abelian, and triangulated categories in algebra and geometry

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The scope of the proposed research project lies within algebraic geometry. Classically algebraic geometry studies algebraic varieties, i.e. the sets of solutions of polynomial equations, using a wide range of methods from commutative algebra. The main motivation for the study are its numerous applications, amongst other in number theory, mathematical physics and string theory.

Recently, various additional categories, such as categories of coherent sheaves and their derived categories, have proved a useful tool in a better understanding of the geometry of varieties. Moreover, these categories are expected to contain a lot of information about varieties which are birationally equivalent to the original variety, i.e. having isomorphic dense open subsets.

With this research project we plan to investigate the relation between derived categories of coherent sheaves and birational geometry. We expect that the basic birational changes, so called flops, are related to an appropriate action of the group  $\mathrm{sl}(2, \mathbb{Z}/4\mathbb{Z})$ . This way we hope to find a connection between birational geometry and the classical theory of modular forms.

We also plan to introduce a new tool in the study of geometry - exact categories. These were originally introduced by Quillen in his study of higher  $K$ -theory. We expect that the detailed study of exact categories and the associated abelian categories would lead to a better understanding of the category of vector bundles on an algebraic variety and the information it contains.